workshop on

Partonic Transverse Momentum Distributions

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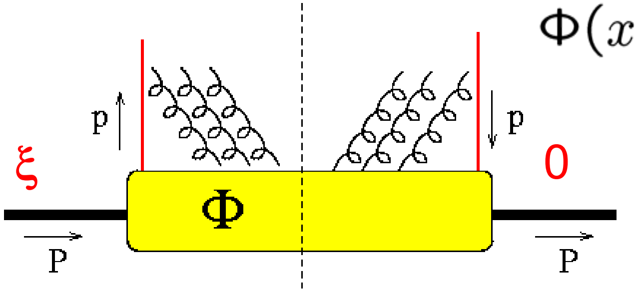
TMD in a spectator diquark model

Marco Radici

Pavia

In collaboration with: A. Bacchetta (Univ. Pavia)
F. Conti (Univ. Pavia)



$$\Phi(x,\mathbf{p}_T,S) = \int \frac{d^4\xi}{(2\pi)^3}\, e^{ip\cdot\xi}\, \langle P,S|\bar{\psi}(0)\, U_{[0,\xi]}\, \psi(\xi)|P,S\rangle$$

$$p \approx (0, xP^+, \mathbf{p}_T) \Rightarrow \xi = (\xi^-, 0, \boldsymbol{\xi}_T)$$

TMD basically unknown !

leading-twist projections

	T-even			T-odd		
TMD	U	L	T	U	L	T
u	f_1					$f_{1T}^{\perp}$
l		g_{1L}	g_{1T}			
t		$h_{1L}^{\perp}$	h_{1T}, $h_{1T}^{\perp}$	$h_1^{\perp}$		

h_1

chiral-odd

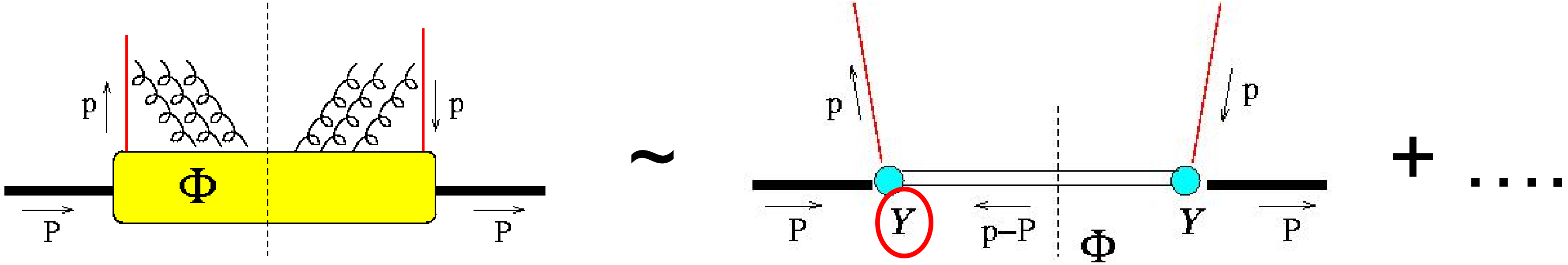
○ known x parametrization
poorly known $\mathbf{p}_T$ “
(gaussian and with no
flavor dependence;
other functional forms are
possible → orbital L)

Hautmann, arXiv:0805.1049 [hep-ph]

“…multi-jet events are potentially sensitive to QCD initial-state radiation that depend on the finite transverse- momentum tail of partonic matrix elements and distributions…”

spectator diquark model : advantages

$$\Phi(x,\mathbf{p}_T,S) = \int \frac{d^4\xi}{(2\pi)^3}\, e^{ip\cdot\xi}\, \langle P,S|\bar{\psi}(0)\, U_{[0,\xi]}\, \psi(\xi)|P,S\rangle \;\approx\; \frac{1}{(2\pi)^3}\,\frac{1}{2(1-x)P^+}\,\overline{\mathcal{M}}^{(0)}(S)\,\mathcal{M}^{(0)}(S) + \dots.$$



$$\mathcal{M}^{(0)}(S) = \langle p-P|\psi(0)|P,S\rangle$$

$$(p-P)^2 = \boxed{M_D^2} \longrightarrow p^2 = \tau(x,\mathbf{p}_T^2) \neq \boxed{m^2}$$

simple, covariant, offshellness, mainly 3 parameters

spectator diquark model : advantages

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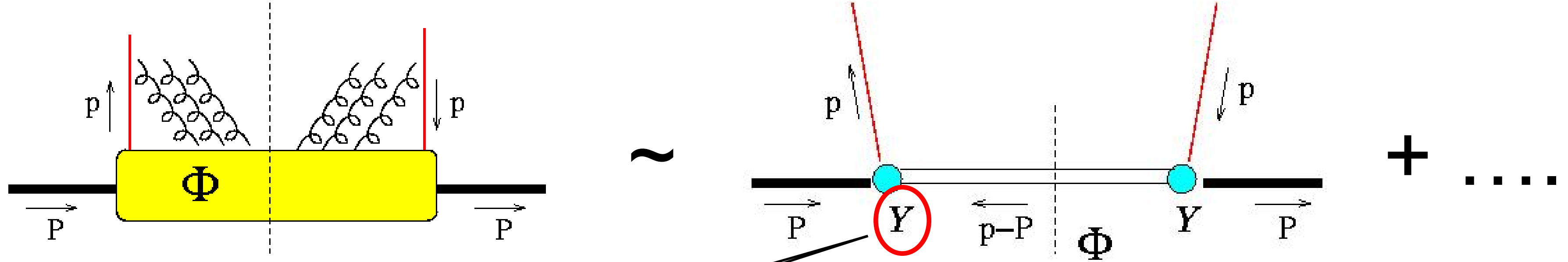

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$\mathcal{Y}$ vertex

N – quark (q) – pointlike diquark (Dq)
(q ½$^+$ in N c.m.)

simple, covariant, offshellness, mainly 3 parameters

spin=0 flavor-singlet [~{ud-du}]

$$\mathcal{Y}_s = i\, g_s(p^2)\, \mathbf{1}$$

spin=1 flavor-triplet [~{dd,ud+du,uu}]

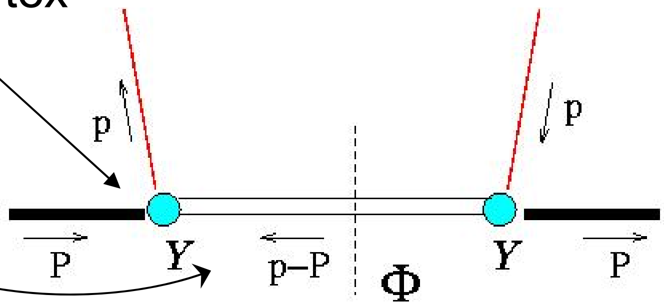
$$\mathcal{Y}_a^\mu = i\,\frac{g_a(p^2)}{\sqrt{2}}\,\gamma^\mu\gamma_5$$

need axial-vector diquarks
to describe *d* in N !

our spectator diquark model : new features

ref. : Bacchetta, Conti, Radici, P.R. D**78**, 074010 (08)

- systematic calculation of ALL leading-twist T-even and T-odd **TMD** → **PDF**
- improvement from model of Jakob, Mulders, Rodrigues, N.P. **A626** (97) 937 and Bacchetta, Schaefer, Yang, P.L. **B578** (04) 109 :
  - ► several functional forms for $g(p^2)$ at N-q-Dq vertex
  - ► several choices of polarization states in the spin=1 diquark propagator $d_{\mu\nu}(p-P)$
  - ► non-gaussian p_T dependence upon x & q
  - ► fix model parameters by fitting low-scale parametrization of $f_1^{u,d}(x)$, $g_1^{u,d}(x)$



- representation of ALL T-even and T-odd TMD as overlaps of **L**ight-**C**one **W**ave **F**unctions (lcwf) including spin=1 Dq's :
  - ► broken SU(4) symmetry of N={q, Dq} system : orbital $L_{q\,Dq} \neq 0$
  - ► universal operator that distinguishes T-even from T-odd overlaps
  - ► generalization to spin=1 Dq of relation between $f_{1T}^{\perp q}$ and κ^q

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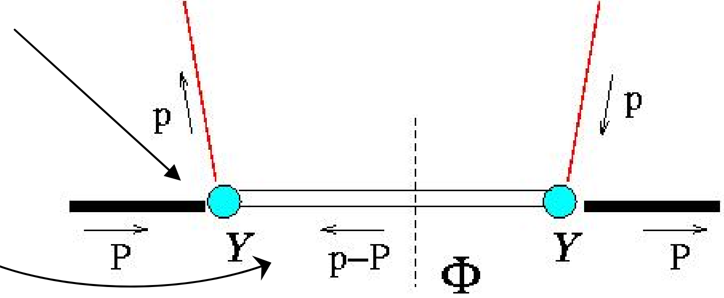

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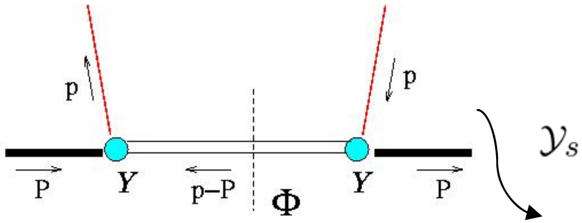
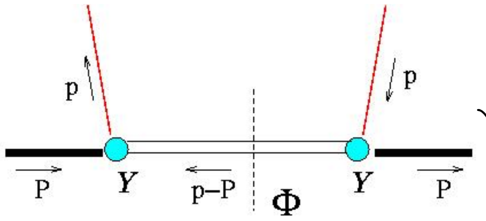
our model cont'ed: T-even TMD



$$\left.\begin{aligned} \mathcal{Y}_s &= i\, g_s(p^2)\, \mathbf{1} \\ \mathcal{Y}_a^\mu &= i\, \frac{g_a(p^2)}{\sqrt{2}}\, \gamma^\mu\, \gamma_5 \end{aligned}\right\} \quad g_D(p^2) = \begin{cases} N_D^{p.l.} & \leftarrow \mathcal{L}(N, q, D) \\ N_D^{dip}\, \frac{p^2 - m^2}{|p^2 - \Lambda_D^2|^2} = N_D^{dip}\, \frac{(p^2-m^2)\,(1-x)^2}{[\mathbf{p}_T^2 + L_D^2(\Lambda_D^2)]^2} & \\ N_D^{exp}\, e^{(p^2-m^2)/\Lambda_D^2} & \end{cases}$$

our model cont'ed: T-even **TMD**



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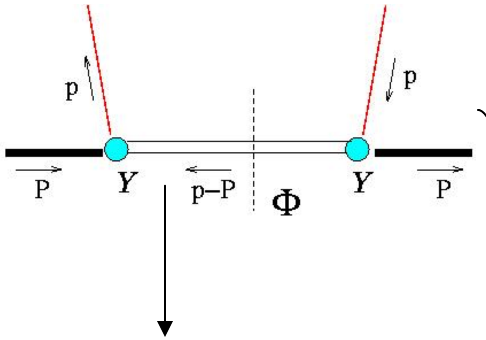
- kill pole $(p^2\text{-}m^2)^{-1}$ and log divergences
- suppresses large $\mathbf{p}_T$
- ~ $(1\text{-}x)^3$ for x→ 1 (Drell-Yan-West)

our model cont'ed: T-even TMD



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spin=1 diquark propagator

- kill pole $(p^2-m^2)^{-1}$ and log divergences
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$$d^{\mu\nu}(p-P) = \begin{cases} -g^{\mu\nu} \\ -g^{\mu\nu} + \frac{P^\mu P^\nu}{M_a^2} \\ -g^{\mu\nu} + \frac{(p-P)^\mu (p-P)^\nu}{M_a^2} \\ -g^{\mu\nu} + \frac{(p-P)^\mu n_-^\nu + (p-P)^\nu n_-^\mu}{(p-P)\cdot n_-} - \frac{M_a^2}{[(p-P)\cdot n_-]^2}\, n_-^\mu n_-^\nu \end{cases}$$

"Feynman" see λ_a = ±, 0, t
Bacchetta, Schaefer, Yang, P.L.**B578** (04) 109

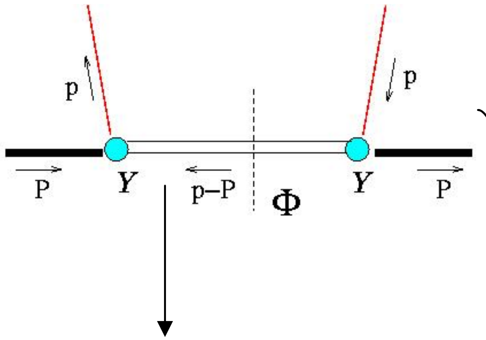
see λ_a = x,y,z only for P at rest
Jakob, Mulders, Rodrigues, N.P.**A626** (97) 937

"covariant" see λ_a = ±, 0
Gamberg, Goldstein, Schlegel, arXiv:0708.0324 [hep-ph]

"light-cone" see λ_a = ±
Brodsky, Hwang, Ma, Schmidt, N.P.**B593** (01) 311

$$d^{\mu\nu}(p-P) = \sum_{\lambda_a} \epsilon^{*\,\mu}_{\lambda_a}(p-P)\, \epsilon^{\nu}_{\lambda_a}(p-P)$$

our model cont'ed: T-even TMD



$$\begin{aligned} \mathcal{Y}_s &= \quad i\, g_s(p^2)\, \mathbf{1} \\ \mathcal{Y}_a^\mu &= i\, \frac{g_a(p^2)}{\sqrt{2}}\, \gamma^\mu \gamma_5 \end{aligned}$$

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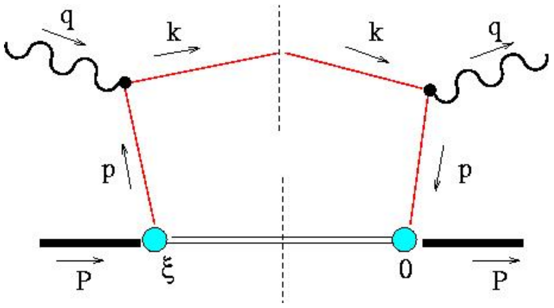
$$d^{\mu\nu}(p-P) = \sum_{\lambda_a} \epsilon_{\lambda_a}^{*\,\mu}(p-P)\, \epsilon_{\lambda_a}^{\nu}(p-P)$$

because…

our model cont'ed: spin=1 diquark propagator

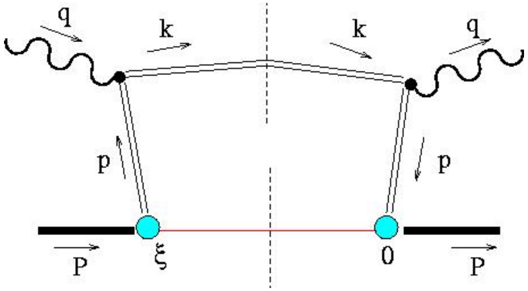
DIS : diquark=charged boson ⇒ not only



spin=0 diquark contributes to F_L only

~~Callan-Gross~~ $\frac{F_L}{F_T} \overset{Q^2\to\infty}{\not\longrightarrow} 0$ but

but also



$$2\,F_1(x) = \sum_q e_q^2\,[f_1^q(x) + f_1^{\bar{q}}(x)]$$

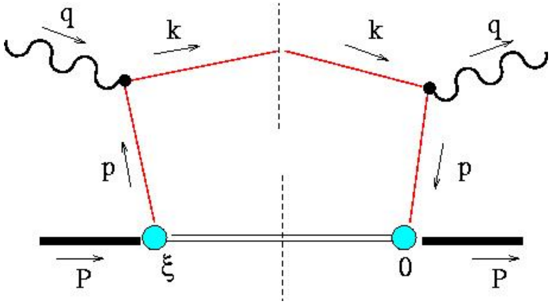
- with "light-cone" $d^{\mu\nu}$, also spin=1 diquark does the same, while with other choices it does not !

- with "Feynman", $d^{\mu\nu}$ propagates unphysical polarization states

$\lambda_a = \pm, 0,$ **t**

our model cont'ed: spin=1 diquark propagator

DIS : diquark=charged boson ⇒ not only



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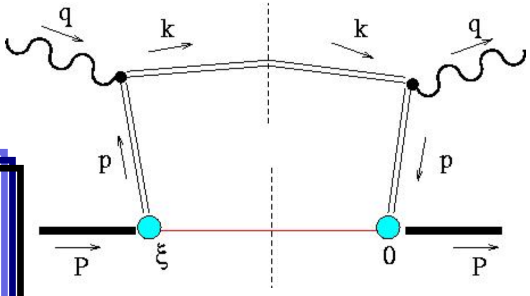

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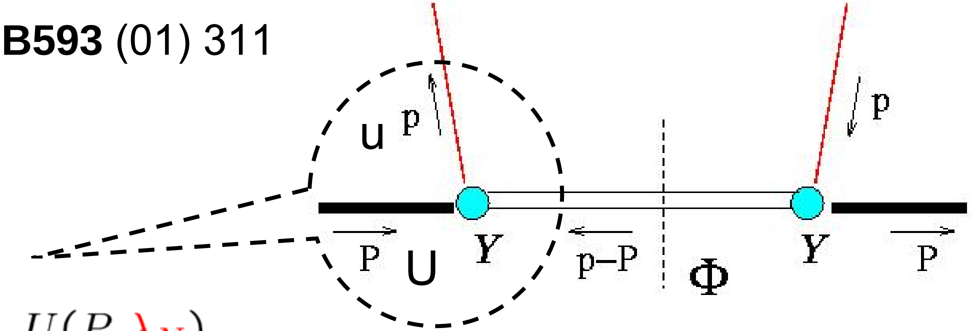
conventions of Lepage, Brodsky, P.R.D**22** (80) 2157

our model cont'ed: LCWF

following Brodksy, Hwang, Ma, Schmidt, N.P.**B593** (01) 311



spin=0 Dq

$$\psi^{\lambda_N}_{\lambda_q}(x,\mathbf{p}_T) = \sqrt{\frac{p^+}{(P-p)^+}}\ \frac{\bar{u}(p,\lambda_q)}{p^2-m^2}\ \mathcal{Y}_s(p^2)\ U(P,\lambda_N)$$

spin=1 Dq

$$\psi^{\lambda_N}_{\lambda_q,\,\lambda_a}(x,\mathbf{p}_T) = \sqrt{\frac{p^+}{(P-p)^+}}\ \frac{\bar{u}(p,\lambda_q)}{p^2-m^2}\ \epsilon^*_\mu(p-P,\lambda_a)\ \mathcal{Y}^\mu_a(p^2)\ U(P,\lambda_N)$$

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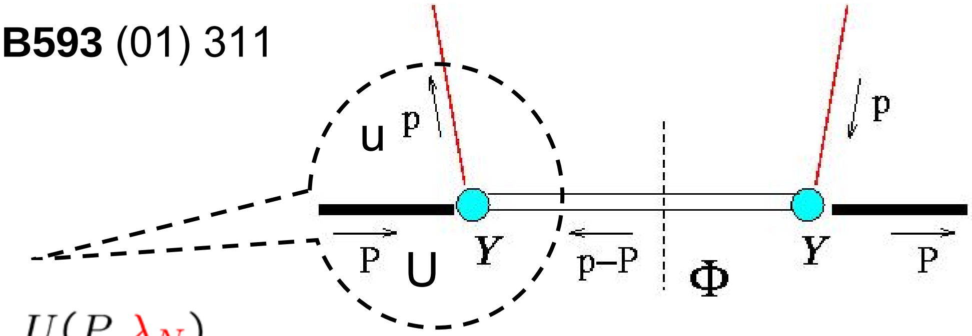

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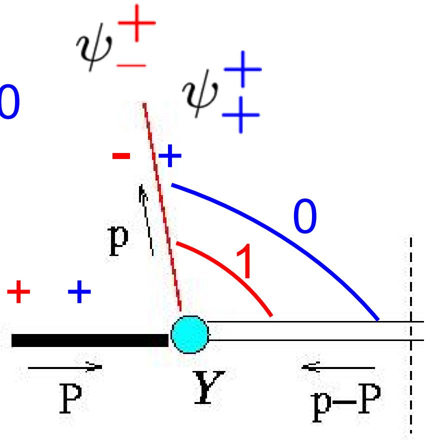
Remarks : $\lambda_q + (\lambda_a) + L_{qDq} = \lambda_N$

► ex: spin=0 Dq=s, λ_N=+, λ_q=- ⇒ L_{qDq}=1
enhanced w.r.t. λ_N=+ , λ_q=+ ⇒ L_{qDq}=0
spin crisis as relativistic effect?

► g.s. of q in N ≠ $½^+$ $\not\Rightarrow$ SU(4)



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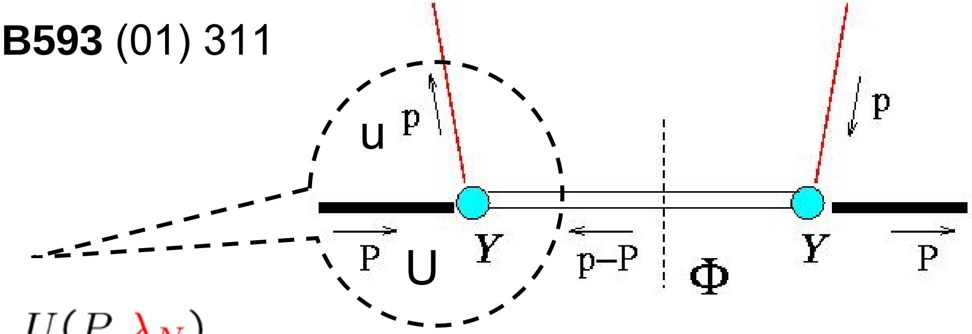

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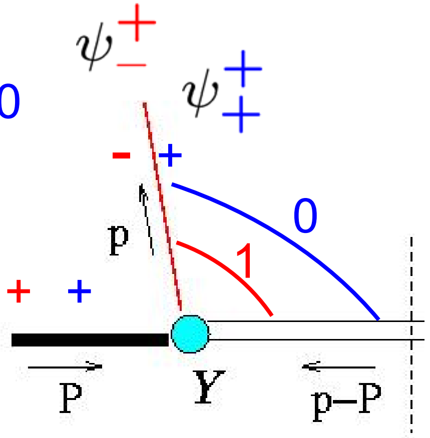

$L_{q\text{-}Dq}$ = 1, 0

our model cont'ed: T-even TMD as LCWF overlaps

$$f_1^s(x,\mathbf{p}_T^2) = \frac{1}{4}\frac{1}{(2\pi)^3}\frac{1}{2(1-x)P^+}\,\mathrm{Tr}\left[\left(\bar{\mathcal{M}}^{(0)}(S)\,\mathcal{M}^{(0)}(S)+\bar{\mathcal{M}}^{(0)}(-S)\,\mathcal{M}^{(0)}(-S)\right)\gamma^+\right]+\mathrm{h.c.}$$

$$= \frac{1}{16\pi^3}\frac{1}{2}\sum_{\lambda_N=\pm}\sum_{\lambda_q=\pm}|\psi^{\lambda_N}_{\lambda_q}|^2 = \frac{N_s^2}{(2\pi)^3}\frac{[\mathbf{p}_T^2+(m+xM)^2]\,(1-x)^3}{2\,(\mathbf{p}_T^2+L_s^2(\Lambda_s^2))^4}$$

$$f_1^a(x,\mathbf{p}_T^2) = \frac{1}{16\pi^3}\frac{1}{2}\sum_{\lambda_N=\pm}\sum_{\lambda_q=\pm,\,\lambda_a=\pm 1}|\psi^{\lambda_N}_{\lambda_q,\,\lambda_a}|^2 = \frac{N_a^2}{(2\pi)^3}\frac{[\mathbf{p}_T^2\,(1+x^2)+(m+xM)^2\,(1-x)^2]\,(1-x)}{2\,(\mathbf{p}_T^2+L_a^2(\Lambda_a^2))^4}$$

$$g_{1L}^D(x,\mathbf{p}_T^2) = \frac{1}{16\pi^3}\sum_{\lambda_D}\left[|\psi^+_{+,\lambda_D}|^2-|\psi^+_{-,\lambda_D}|^2\right]$$

$$L_D^2(m^2)=xM_D^2+(1-x)m^2-x(1-x)M^2$$

our model cont'ed: T-even **TMD** as LCWF overlaps

$$\begin{aligned} f_1^s(x,\mathbf{p}_T^2) &= \frac{1}{4}\frac{1}{(2\pi)^3}\frac{1}{2(1-x)P^+}\,\mathrm{Tr}\left[\left(\bar{\mathcal{M}}^{(0)}(S)\,\mathcal{M}^{(0)}(S)+\bar{\mathcal{M}}^{(0)}(-S)\,\mathcal{M}^{(0)}(-S)\right)\,\gamma^+\right]+\mathrm{h.c.} \\ &= \frac{1}{16\pi^3}\frac{1}{2}\sum_{\lambda_N=\pm}\sum_{\lambda_q=\pm}|\psi^{\lambda_N}_{\lambda_q}|^2=\frac{N_s^2}{(2\pi)^3}\frac{[\mathbf{p}_T^2+(m+xM)^2]\,(1-x)^3}{2\,(\mathbf{p}_T^2+L_s^2(\Lambda_s^2))^4} \end{aligned}$$

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$$g_{1L}^D(x,\mathbf{p}_T^2) = \frac{1}{16\pi^3}\sum_{\lambda_D}\left[|\psi^+_{+,\lambda_D}|^2-|\psi^+_{-,\lambda_D}|^2\right]$$

$$L_D^2(m^2)=xM_D^2+(1-x)m^2-x(1-x)M^2$$

non-gaussian $\mathbf{p}_T$ dependence
unfactorized from x
depends on flavor

fixing model parameters

Dq model of Jakob, Mulders, Rodrigues, N.P. **A626** (97) 937

s = (spin=0 isospin=0)

a = (spin=1 isospin=0)

SU(4) of |p> ⇒

$$f_1^u = \frac{3}{2} f_1^s + \frac{1}{2} f_1^a$$

$$f_1^d = f_1^a$$

a' = (spin=1 isospin=1)

~~SU~~(4) of |p> ⇒

model parameters

$$f_1^u = c_s^2\, f_1^s + c_a^2\, f_1^a$$

$$f_1^d = c_{a'}^2\, f_1^{a'}$$

Parameters: m = M/3 ; N_s , N_a , $N_{a'}$; M_s , M_a , $M_{a'}$; Λ_s , Λ_a , $\Lambda_{a'}$; c_s , c_a , $c_{a'}$

fixing model parameters

Dq model of Jakob, Mulders, Rodrigues, N.P. **A626** (97) 937

s = (spin=0 isospin=0)

a = (spin=1 isospin=0)

SU(4) of |p> ⇒

$$\begin{aligned} f_1^u &= \frac{3}{2} f_1^s + \frac{1}{2} f_1^a \\ f_1^d &= f_1^a \end{aligned}$$

a' = (spin=1 isospin=1)

~~SU~~(4) of |p> ⇒

model parameters

$$\begin{aligned} f_1^u &= c_s^2 \, f_1^s + c_a^2 \, f_1^a \\ f_1^d &= c_{a'}^2 \, f_1^{a'} \end{aligned}$$

Parameters: $m = M/3$; $\underbrace{N_s , N_a , N_{a'}}_{\text{fixed by}}$; $M_s , M_a , M_{a'}$; $\Lambda_s , \Lambda_a , \Lambda_{a'}$; $c_s , c_a , c_{a'}$

fixed by

$||f_1^2|| = ||f_1^a|| = ||f_1^{a'}|| =1$

fixing model parameters

Dq model of Jakob, Mulders, Rodrigues, N.P. **A626** (97) 937

s = (spin=0 isospin=0)

a = (spin=1 isospin=0)

SU(4) of |p> ⇒

$$\begin{aligned} f_1^u &= \frac{3}{2} f_1^s + \frac{1}{2} f_1^a \\ f_1^d &= f_1^a \end{aligned}$$

a' = (spin=1 isospin=1)

$\cancel{\text{SU}}$(4) of |p> ⇒

model parameters

$$\begin{aligned} f_1^u &= c_s^2\, f_1^s + c_a^2\, f_1^a \\ f_1^d &= c_{a'}^2\, f_1^{a'} \end{aligned}$$

Parameters: $m = M/3\ ;\ \underbrace{N_s , N_a , N_{a'}}\ ;\ \underbrace{M_s , M_a , M_{a'}\ ;\ \Lambda_s , \Lambda_a , \Lambda_{a'}\ ;\ c_s , c_a , c_{a'}}$

fixed by

$||f_1^2|| = ||f_1^a|| = ||f_1^{a'}|| =1$

9 parameters fixed by fitting

$f_1^u(x)$, $f_1^d(x)$, $g_1^u(x)$, $g_1^d(x)$

at lowest possible scale:

$f_1^u(x)$, $f_1^d(x)$ at Q^2 = 0.3 GeV^2

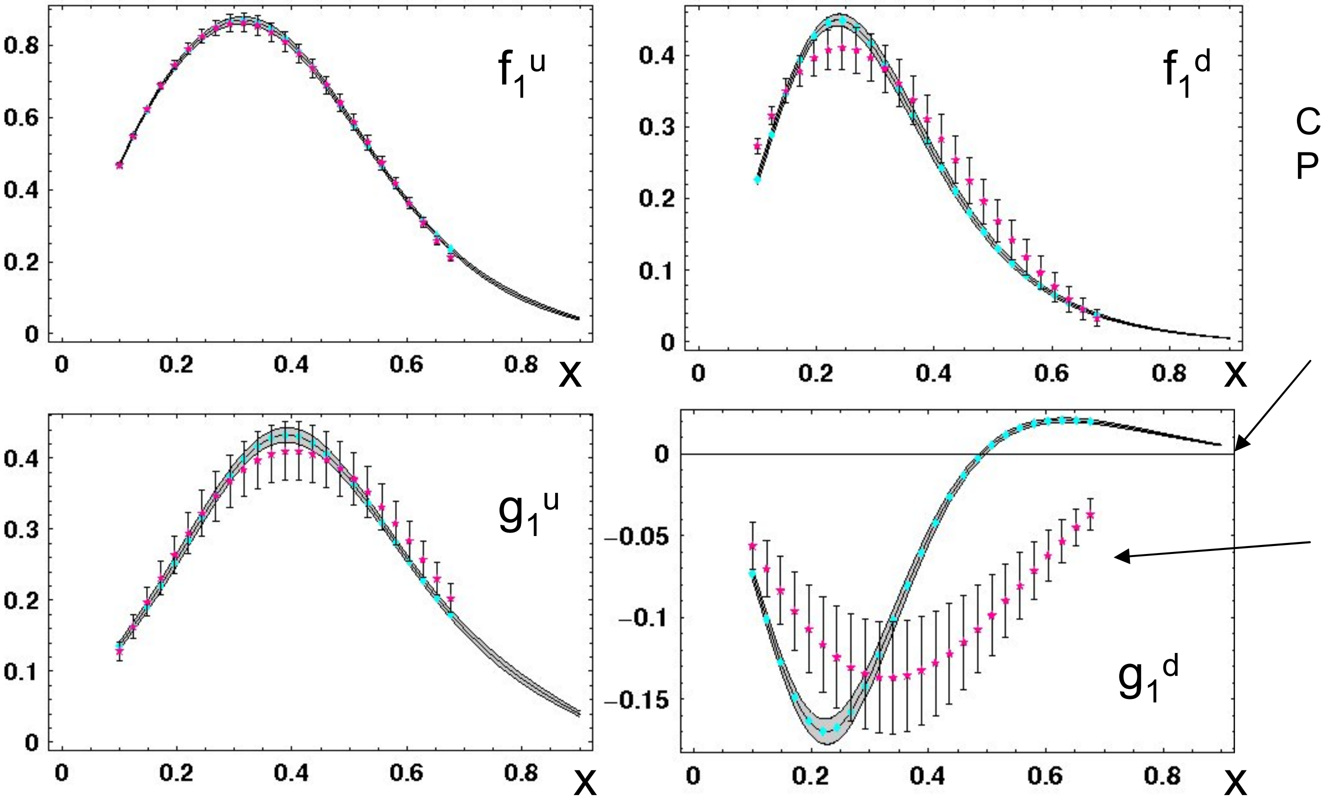
ZEUS, P.R.D**67** (03) 012007

$g_1^u(x)$, $g_1^d(x)$ at Q^2=0.26 GeV^2

GRSV01 at LO

hadronic scale of the model

Q_0^2 ~ 0.3 GeV^2



χ^2 / d.o.f. = 3.88

Chekanov *et al*. (ZEUS),
P.R.D**67** (03) 012007

our fit; error band from MINUIT covariant error matrix

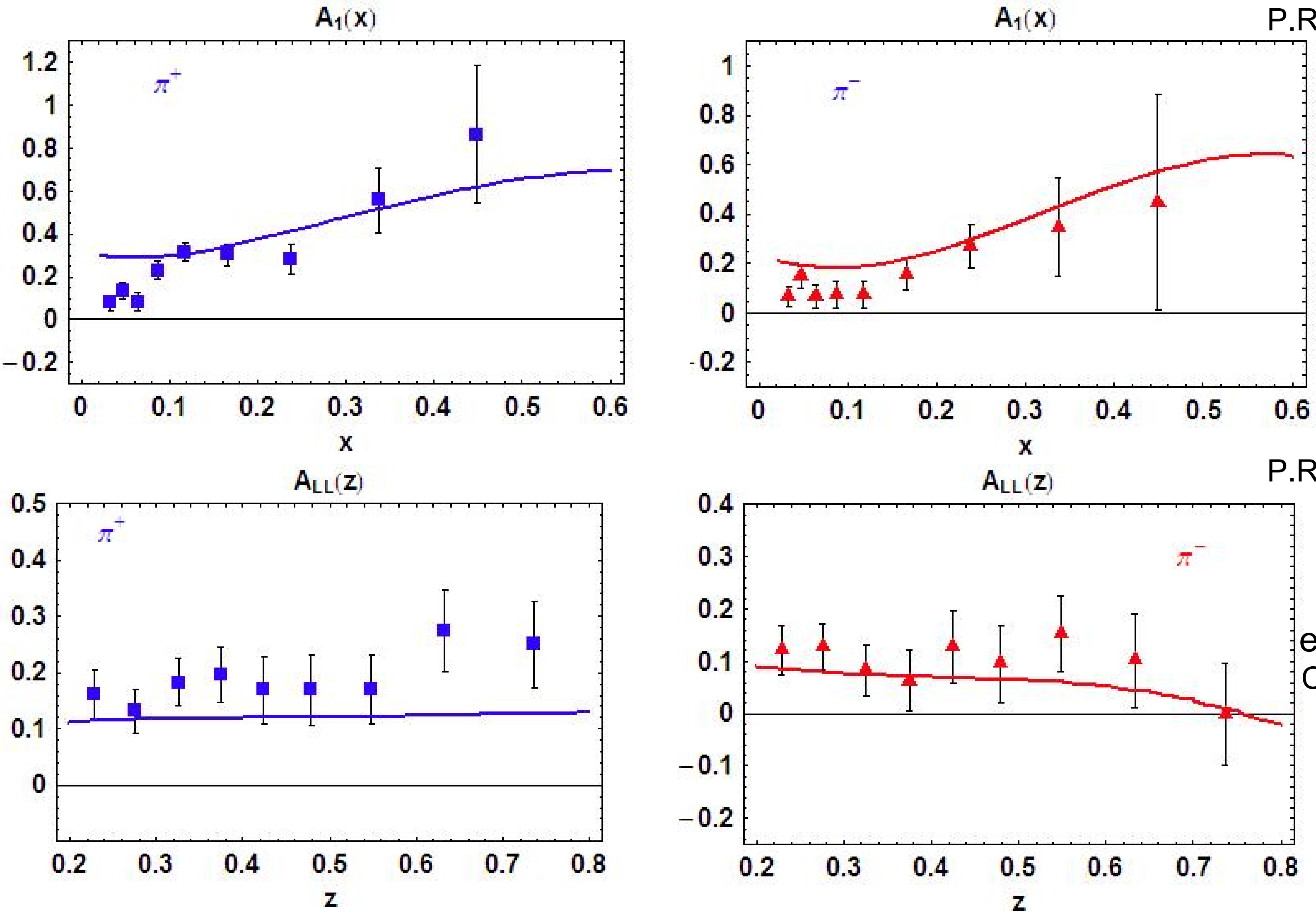
phenomenological parametrizations

GRSV01 at LO

Dq	M_D	Λ_D	c_D
s	0.822 ± 0.053	0.609 ± 0.038	0.847 ± 0.111
a	1.492 ± 0.173	0.716 ± 0.074	1.061 ± 0.085
a’	0.890 ± 0.008	0.376 ± 0.005	0.880 ± 0.008

$$P_q = \int_0^1 dx\; x\; (f_1^u + f_1^d) = 0.584 \pm 0.010$$
$$\text{ZEUS} = 0.55$$

$$g_A = \int_0^1 dx\; (g_1^u - g_1^d) = 0.966 \pm 0.038$$
$$\text{GRSV01} = 0.969 \pm 0.096$$



HERMES data
P.R.D**71**,012003(05)

s=56.2 GeV^2
<Q^2>=2.5 “
$y_{min}(x)<y<0.85$
$0.023<x<0.4$

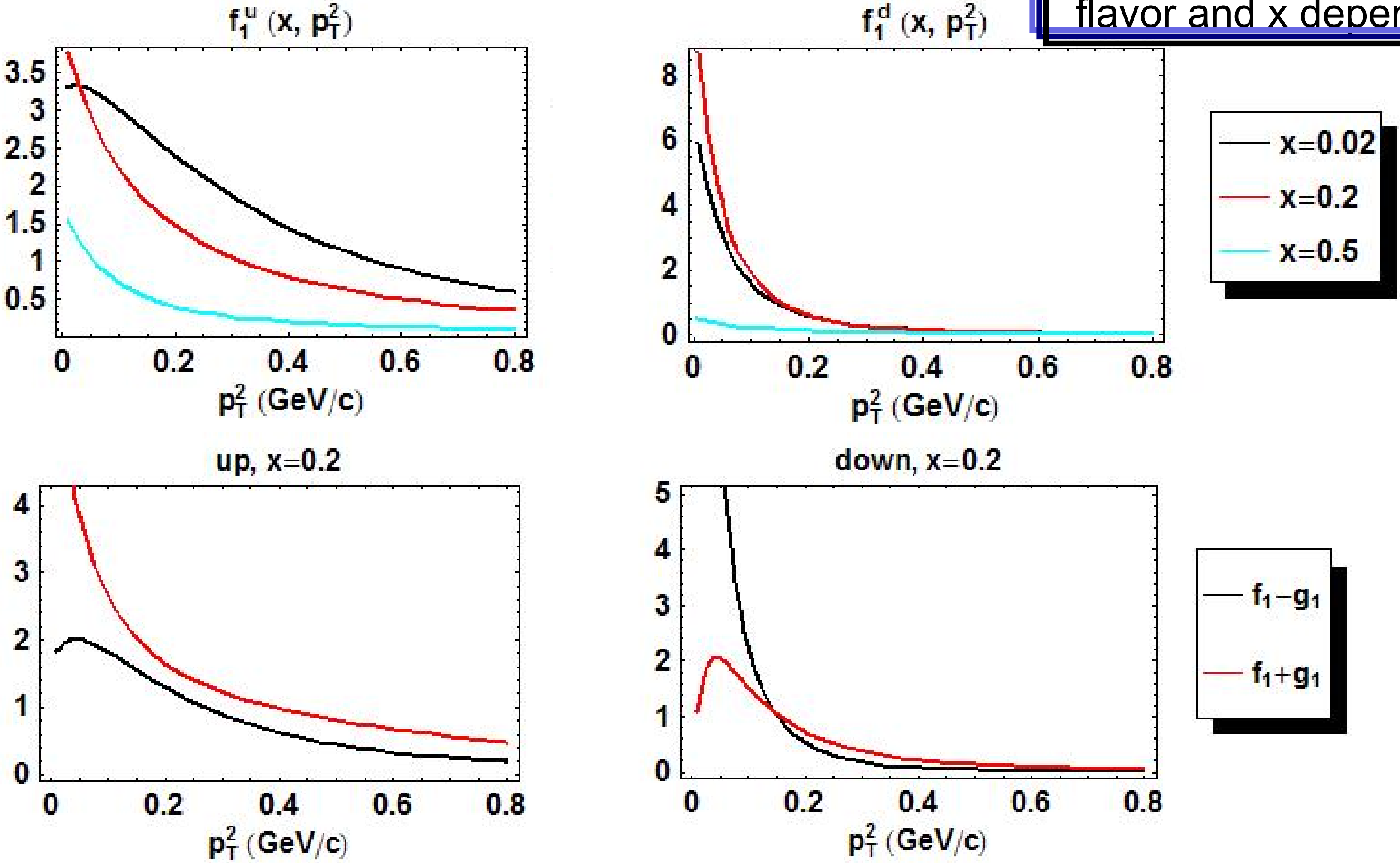
$D_1 \rightarrow$ DSS
P.R.D**75**,114010(07)
fav.+ unfav.

model PDF
evol.→ Kumano
C.P.C. **94**,185(96)

$$A_{LL}(x,z,Q^2) = \frac{C(y(x))}{A(y(x))} \frac{\sum_q e_q^2 \; x g_1^q(x,Q^2(x)) \; D_1(z,Q^2(x))}{\sum_q e_q^2 \; x f_1^q(x,Q^2(x)) \; D_1(z,Q^2(x))} = \frac{C(y)}{A(y)} \; A_1(x,z,Q^2)$$

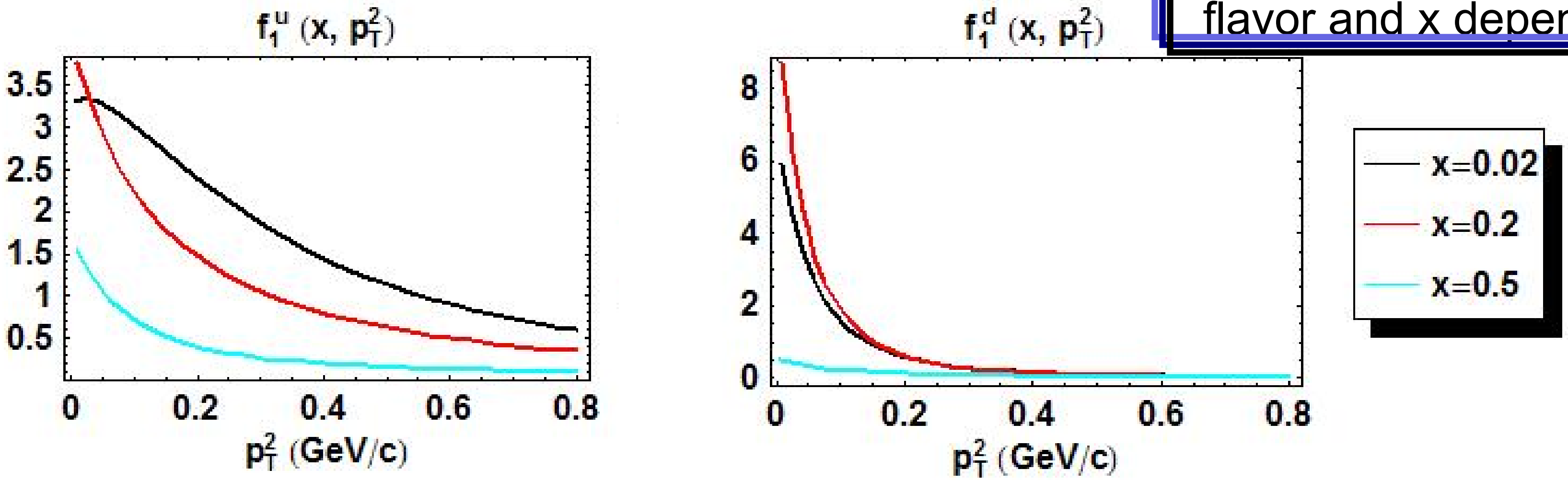
p_T model dependence

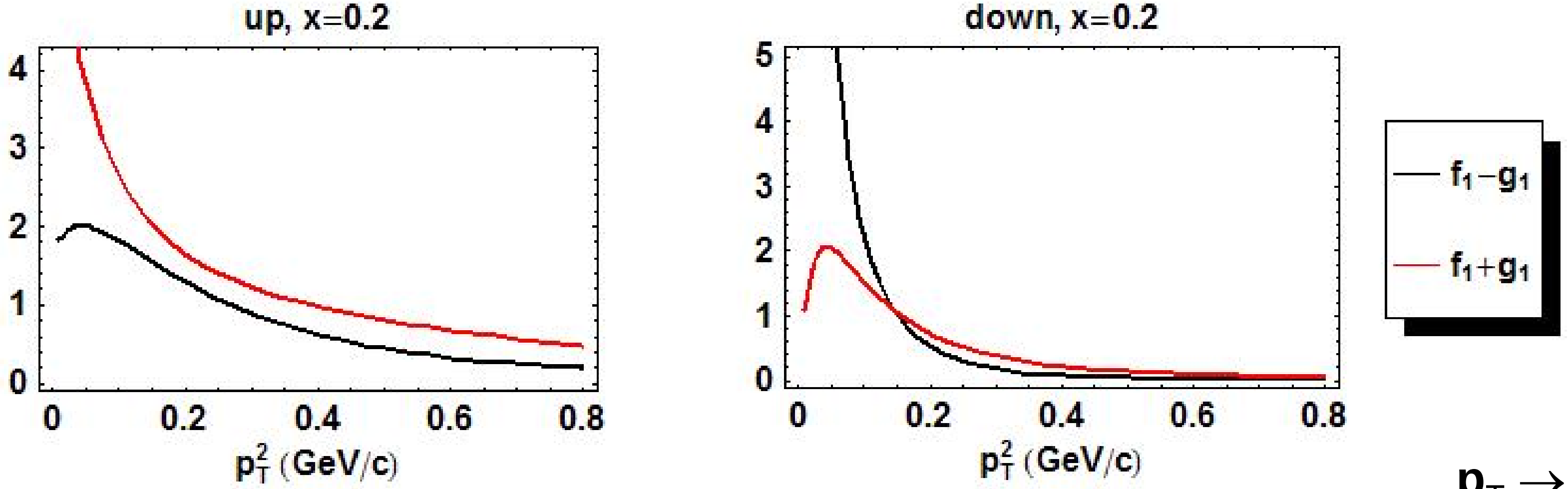
flavor and x dependence

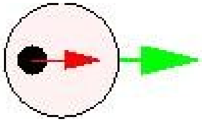
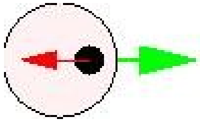


p_T model dependence

flavor and x dependence





f_1+g_1 ~  f_1-g_1 ~ 

p={d (uu)} $\psi^+_{+\pm}$ has $L_{q\text{-}Dq}$ =1 $\xrightarrow{p_T \to 0}$ 0

ψ^+_{-+} $L_{q\text{-}Dq}$ =0 does not

p={u (ud)} mix with ψ^+_+(L=0) and ψ^+_-(L=1)

our model cont'ed : ⊥-pol. T-even **TMD** as LCWF overlaps

need to define the state with polarization along $\hat{\mathbf{s}}_T = (\cos\phi, \sin\phi)$

$$U(P,\uparrow) = \frac{1}{\sqrt{2}}\left(U(P,+) + e^{i\phi}\,U(P,-)\right)$$

$$U(P,\downarrow) = \frac{1}{\sqrt{2}}\left(U(P,+) + e^{i(\phi+\pi)}\,U(P,-)\right)$$

$$\psi^{\lambda_N}_{\lambda_q} = \sqrt{\frac{p^+}{(P-p)^+}}\,\frac{\bar{u}(p,\lambda_q)}{p^2-m^2}\,\mathcal{Y}_s(p^2)\,U(P,\lambda_N)$$

agree with Barone, Ratcliffe, *Transverse Spin Physics*
(World Scientific, Singapore, 2003)

D = s $\lambda_s = \varnothing$
D = a $\lambda_a = \pm$

$$\frac{\mathbf{p}_T\cdot\hat{\mathbf{s}}_T}{M}\,g_{1T}^{D}(x,\mathbf{p}_T^2) = \frac{1}{16\pi^3}\sum_{\lambda_D}\left[|\psi^{\uparrow}_{+,\lambda_D}|^2 - |\psi^{\uparrow}_{-,\lambda_D}|^2\right]$$

$$\frac{\mathbf{p}_T\cdot\hat{\mathbf{s}}_{qT}}{M}\,h_{1L}^{\perp D}(x,\mathbf{p}_T^2) = \frac{1}{16\pi^3}\sum_{\lambda_D}\left[|\psi^{+}_{\uparrow,\lambda_D}|^2 - |\psi^{+}_{\downarrow,\lambda_D}|^2\right]$$

$$\frac{\hat{\mathbf{s}}_T\cdot\hat{\mathbf{s}}_{qT}}{M}\,h_{1T}^{D}(x,\mathbf{p}_T^2) + \frac{\mathbf{p}_T\cdot\hat{\mathbf{s}}_T}{M}\,\frac{\mathbf{p}_T\cdot\hat{\mathbf{s}}_{qT}}{M}\,h_{1T}^{\perp D}(x,\mathbf{p}_T^2) = \frac{1}{16\pi^3}\sum_{\lambda_D}\left[|\psi^{\uparrow}_{\uparrow,\lambda_D}|^2 - |\psi^{\uparrow}_{\downarrow,\lambda_D}|^2\right]$$

our model cont'ed: T-even list

Bacchetta, Conti, Radici, P.R. D**78**, 074010 (08)

TMD

$$g_{1L}^{s}(x,p_T^2) = \frac{N_s^2}{(2\pi)^3}\,\frac{\left[-p_T^2+(m+xM)^2\right](1-x)^3}{2\,(p_T^2+L_s^2)^4}\,,$$

$$g_{1L}^{a}(x,p_T^2) = \frac{N_a^2}{(2\pi)^3}\,\frac{\left[p_T^2\,(1+x^2)-(m+xM)^2\,(1-x)^2\right](1-x)}{2\,(p_T^2+L_a^2)^4}\,,$$

$$g_{1T}^{s}(x,p_T^2) = \frac{N_s^2}{(2\pi)^3}\,\frac{M\,(m+xM)\,(1-x)^3}{(p_T^2+L_s^2)^4}\,,$$

$$g_{1T}^{a}(x,p_T^2) = \frac{N_a^2}{(2\pi)^3}\,\frac{xM\,(m+xM)\,(1-x)^2}{(p_T^2+L_a^2)^4}\,,$$

$$h_{1L}^{\perp s}(x,p_T^2) = -\frac{N_s^2}{(2\pi)^3}\,\frac{M\,(m+xM)\,(1-x)^3}{(p_T^2+L_s^2)^4}\,,$$

$$h_{1L}^{\perp a}(x,p_T^2) = \frac{N_a^2}{(2\pi)^3}\,\frac{M\,(m+xM)\,(1-x)^2}{(p_T^2+L_a^2)^4}\,,$$

$$h_{1T}^{s}(x,p_T^2) = \frac{N_s^2}{(2\pi)^3}\,\frac{\left[p_T^2+(m+xM)^2\right](1-x)^3}{2\,(p_T^2+L_s^2)^4}\,,$$

$$h_{1T}^{a}(x,p_T^2) = -\frac{N_a^2}{(2\pi)^3}\,\frac{p_T^2\,x(1-x)}{(p_T^2+L_a^2)^4}\,,$$

$$h_{1T}^{\perp s}(x,p_T^2) = -\frac{N_s^2}{(2\pi)^3}\,\frac{M^2\,(1-x)^3}{(p_T^2+L_s^2)^4}\,,$$

$$h_{1T}^{\perp a}(x,p_T^2) = 0\,.$$

PDF

$$f_1^{s}(x) = \frac{N_s^2}{(2\pi)^2}\,\frac{\left[L_s^2(\Lambda_s^2)+2\,(m+xM)^2\right](1-x)^3}{24\,L_s^6(\Lambda_s^2)}$$

$$f_1^{a}(x) = \frac{N_a^2}{(2\pi)^2}\,\frac{\left[L_a^2(\Lambda_a^2)\,(1+x^2)+2\,(m+xM)^2\,(1-x)^2\right](1-x)}{24\,L_a^6(\Lambda_a^2)}$$

$$g_1^{s}(x) = \frac{N_s^2}{(2\pi)^2}\,\frac{\left[2\,(m+xM)^2-L_s^2(\Lambda_s^2)\right](1-x)^3}{24\,L_s^6(\Lambda_s^2)}$$

$$g_1^{a}(x) = -\frac{N_a^2}{(2\pi)^2}\,\frac{\left[2\,(m+xM)^2\,(1-x)^2-(1+x^2)\,L_a^2(\Lambda_a^2)\right](1-x)}{24\,L_a^6(\Lambda_a^2)}$$

$$h_1^{s}(x) = \frac{N_s^2}{(2\pi)^2}\,\frac{(m+xM)^2\,(1-x)^3}{12\,L_s^6(\Lambda_s^2)}$$

$$h_1^{a}(x) = -\frac{N_a^2}{(2\pi)^2}\,\frac{x(1-x)}{12\,L_a^4(\Lambda_a^2)}\,.$$

our model cont'ed: T-even list

Bacchetta, Conti, Radici, P.R. D**78**, 074010 (08)

TMD

$$g_{1L}^{s}(x,p_T^2)=\frac{N_s^2}{(2\pi)^3}\,\frac{\left[-p_T^2+(m+xM)^2\right](1-x)^3}{2\,(p_T^2+L_s^2)^4}\,,$$

$$g_{1L}^{a}(x,p_T^2)=\frac{N_a^2}{(2\pi)^3}\,\frac{\left[p_T^2\,(1+x^2)-(m+xM)^2\,(1-x)^2\right](1-x)}{2\,(p_T^2+L_a^2)^4}\,,$$

$$g_{1T}^{s}(x,p_T^2)=\frac{N_s^2}{(2\pi)^3}\,\frac{M\,(m+xM)\,(1-x)^3}{(p_T^2+L_s^2)^4}\,,$$

$$g_{1T}^{a}(x,p_T^2)=\frac{N_a^2}{(2\pi)^3}\,\frac{xM\,(m+xM)\,(1-x)^2}{(p_T^2+L_a^2)^4}\,,$$

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$$h_{1T}^{s}(x,p_T^2)=\frac{N_s^2}{(2\pi)^3}\,\frac{\left[p_T^2+(m+xM)^2\right](1-x)^3}{2\,(p_T^2+L_s^2)^4}\,,$$

$$h_{1T}^{a}(x,p_T^2)=-\frac{N_a^2}{(2\pi)^3}\,\frac{p_T^2\,x(1-x)}{(p_T^2+L_a^2)^4}\,,$$

$$h_{1T}^{\perp s}(x,p_T^2)=-\frac{N_s^2}{(2\pi)^3}\,\frac{M^2\,(1-x)^3}{(p_T^2+L_s^2)^4}\,,$$

$$\boxed{h_{1T}^{\perp a}(x,p_T^2)=0\,.}$$

PDF

$$f_1^{s}(x)=\frac{N_s^2}{(2\pi)^2}\,\frac{\left[L_s^2(\Lambda_s^2)+2\,(m+xM)^2\right](1-x)^3}{24\,L_s^6(\Lambda_s^2)}$$

$$f_1^{a}(x)=\frac{N_a^2}{(2\pi)^2}\,\frac{\left[L_a^2(\Lambda_a^2)\,(1+x^2)+2\,(m+xM)^2\,(1-x)^2\right](1-x)}{24\,L_a^6(\Lambda_a^2)}$$

$$g_1^{s}(x)=\frac{N_s^2}{(2\pi)^2}\,\frac{\left[2\,(m+xM)^2-L_s^2(\Lambda_s^2)\right](1-x)^3}{24\,L_s^6(\Lambda_s^2)}$$

$$g_1^{a}(x)=-\frac{N_a^2}{(2\pi)^2}\,\frac{\left[2\,(m+xM)^2\,(1-x)^2-(1+x^2)\,L_a^2(\Lambda_a^2)\right](1-x)}{24\,L_a^6(\Lambda_a^2)}$$

$$h_1^{s}(x)=\frac{N_s^2}{(2\pi)^2}\,\frac{(m+xM)^2\,(1-x)^3}{12\,L_s^6(\Lambda_s^2)}$$

$$h_1^{a}(x)=-\frac{N_a^2}{(2\pi)^2}\,\frac{x(1-x)}{12\,L_a^4(\Lambda_a^2)}\,.$$

s

$$g_{1L}(x,\mathbf{p}_T^2)-h_1(x,\mathbf{p}_T^2)=\frac{\mathbf{p}_T^2}{2M^2}\,h_{1T}^{\perp}(x,\mathbf{p}_T^2)$$

Pretzelosity
Avakian *et al.*
P.R.D**78**, 114024 (08)

our model cont'ed: T-even list

Bacchetta, Conti, Radici, P.R. D**78**, 074010 (08)

TMD

$$g_{1L}^{s}(x,p_T^2) = \frac{N_s^2}{(2\pi)^3}\,\frac{\left[-p_T^2+(m+xM)^2\right](1-x)^3}{2\,(p_T^2+L_s^2)^4}\,,$$

$$g_{1L}^{a}(x,p_T^2) = \frac{N_a^2}{(2\pi)^3}\,\frac{\left[p_T^2\,(1+x^2)-(m+xM)^2\,(1-x)^2\right](1-x)}{2\,(p_T^2+L_a^2)^4}\,,$$

$$g_{1T}^{s}(x,p_T^2) = \frac{N_s^2}{(2\pi)^3}\,\frac{M\,(m+xM)\,(1-x)^3}{(p_T^2+L_s^2)^4}\,,$$

$$g_{1T}^{a}(x,p_T^2) = \frac{N_a^2}{(2\pi)^3}\,\frac{xM\,(m+xM)\,(1-x)^2}{(p_T^2+L_a^2)^4}\,,$$

$$h_{1L}^{\perp s}(x,p_T^2) = -\frac{N_s^2}{(2\pi)^3}\,\frac{M\,(m+xM)\,(1-x)^3}{(p_T^2+L_s^2)^4}\,,$$

$$h_{1L}^{\perp a}(x,p_T^2) = \frac{N_a^2}{(2\pi)^3}\,\frac{M\,(m+xM)\,(1-x)^2}{(p_T^2+L_a^2)^4}\,,$$

$$h_{1T}^{s}(x,p_T^2) = \frac{N_s^2}{(2\pi)^3}\,\frac{\left[p_T^2+(m+xM)^2\right](1-x)^3}{2\,(p_T^2+L_s^2)^4}\,,$$

$$h_{1T}^{a}(x,p_T^2) = -\frac{N_a^2}{(2\pi)^3}\,\frac{p_T^2\,x(1-x)}{(p_T^2+L_a^2)^4}\,,$$

$$h_{1T}^{\perp s}(x,p_T^2) = -\frac{N_s^2}{(2\pi)^3}\,\frac{M^2\,(1-x)^3}{(p_T^2+L_s^2)^4}\,,$$

$$\boxed{h_{1T}^{\perp a}(x,p_T^2) = 0\,.}$$

PDF

$$f_1^{s}(x) = \frac{N_s^2}{(2\pi)^2}\,\frac{\left[L_s^2(\Lambda_s^2)+2\,(m+xM)^2\right](1-x)^3}{24\,L_s^6(\Lambda_s^2)}$$

$$f_1^{a}(x) = \frac{N_a^2}{(2\pi)^2}\,\frac{\left[L_a^2(\Lambda_a^2)\,(1+x^2)+2\,(m+xM)^2\,(1-x)^2\right](1-x)}{24\,L_a^6(\Lambda_a^2)}$$

$$g_1^{s}(x) = \frac{N_s^2}{(2\pi)^2}\,\frac{\left[2\,(m+xM)^2-L_s^2(\Lambda_s^2)\right](1-x)^3}{24\,L_s^6(\Lambda_s^2)}$$

$$g_1^{a}(x) = -\frac{N_a^2}{(2\pi)^2}\,\frac{\left[2\,(m+xM)^2\,(1-x)^2-(1+x^2)\,L_a^2(\Lambda_a^2)\right](1-x)}{24\,L_a^6(\Lambda_a^2)}$$

$$h_1^{s}(x) = \frac{N_s^2}{(2\pi)^2}\,\frac{(m+xM)^2\,(1-x)^3}{12\,L_s^6(\Lambda_s^2)}$$

$$h_1^{a}(x) = -\frac{N_a^2}{(2\pi)^2}\,\frac{x(1-x)}{12\,L_a^4(\Lambda_a^2)}\,.$$

a

$$\boxed{g_{1L}(x,\mathrm{p}_T^2)-h_1(x,\mathrm{p}_T^2) \neq \frac{\mathrm{p}_T^2}{2M^2}\,h_{1T}^{\perp}(x,\mathrm{p}_T^2)}$$

Pretzelosity
Avakian *et al.*
P.R.D**78**, 114024 (08)

our model cont'ed: T-even list

Bacchetta, Conti, Radici, P.R. D**78**, 074010 (08)

TMD

$$g_{1L}^{s}(x,p_T^2) = \frac{N_s^2}{(2\pi)^3}\,\frac{\left[-p_T^2+(m+xM)^2\right](1-x)^3}{2\,(p_T^2+L_s^2)^4}\,,$$

$$g_{1L}^{a}(x,p_T^2) = \frac{N_a^2}{(2\pi)^3}\,\frac{\left[p_T^2\,(1+x^2)-(m+xM)^2\,(1-x)^2\right](1-x)}{2\,(p_T^2+L_a^2)^4}\,,$$

$$g_{1T}^{s}(x,p_T^2) = \frac{N_s^2}{(2\pi)^3}\,\frac{M\,(m+xM)\,(1-x)^3}{(p_T^2+L_s^2)^4}\,,$$

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$$\boxed{h_{1T}^{\perp a}(x,p_T^2) = 0\,.}$$

PDF

$$f_1^{s}(x) = \frac{N_s^2}{(2\pi)^2}\,\frac{\left[L_s^2(\Lambda_s^2)+2\,(m+xM)^2\right](1-x)^3}{24\,L_s^6(\Lambda_s^2)}$$

$$f_1^{a}(x) = \frac{N_a^2}{(2\pi)^2}\,\frac{\left[L_a^2(\Lambda_a^2)\,(1+x^2)+2\,(m+xM)^2\,(1-x)^2\right](1-x)}{24\,L_a^6(\Lambda_a^2)}$$

$$g_1^{s}(x) = \frac{N_s^2}{(2\pi)^2}\,\frac{\left[2\,(m+xM)^2-L_s^2(\Lambda_s^2)\right](1-x)^3}{24\,L_s^6(\Lambda_s^2)}$$

$$g_1^{a}(x) = -\frac{N_a^2}{(2\pi)^2}\,\frac{\left[2\,(m+xM)^2\,(1-x)^2-(1+x^2)\,L_a^2(\Lambda_a^2)\right](1-x)}{24\,L_a^6(\Lambda_a^2)}$$

$$h_1^{s}(x) = \frac{N_s^2}{(2\pi)^2}\,\frac{(m+xM)^2\,(1-x)^3}{12\,L_s^6(\Lambda_s^2)}$$

$$h_1^{a}(x) = -\frac{N_a^2}{(2\pi)^2}\,\frac{x(1-x)}{12\,L_a^4(\Lambda_a^2)}\,.$$

a

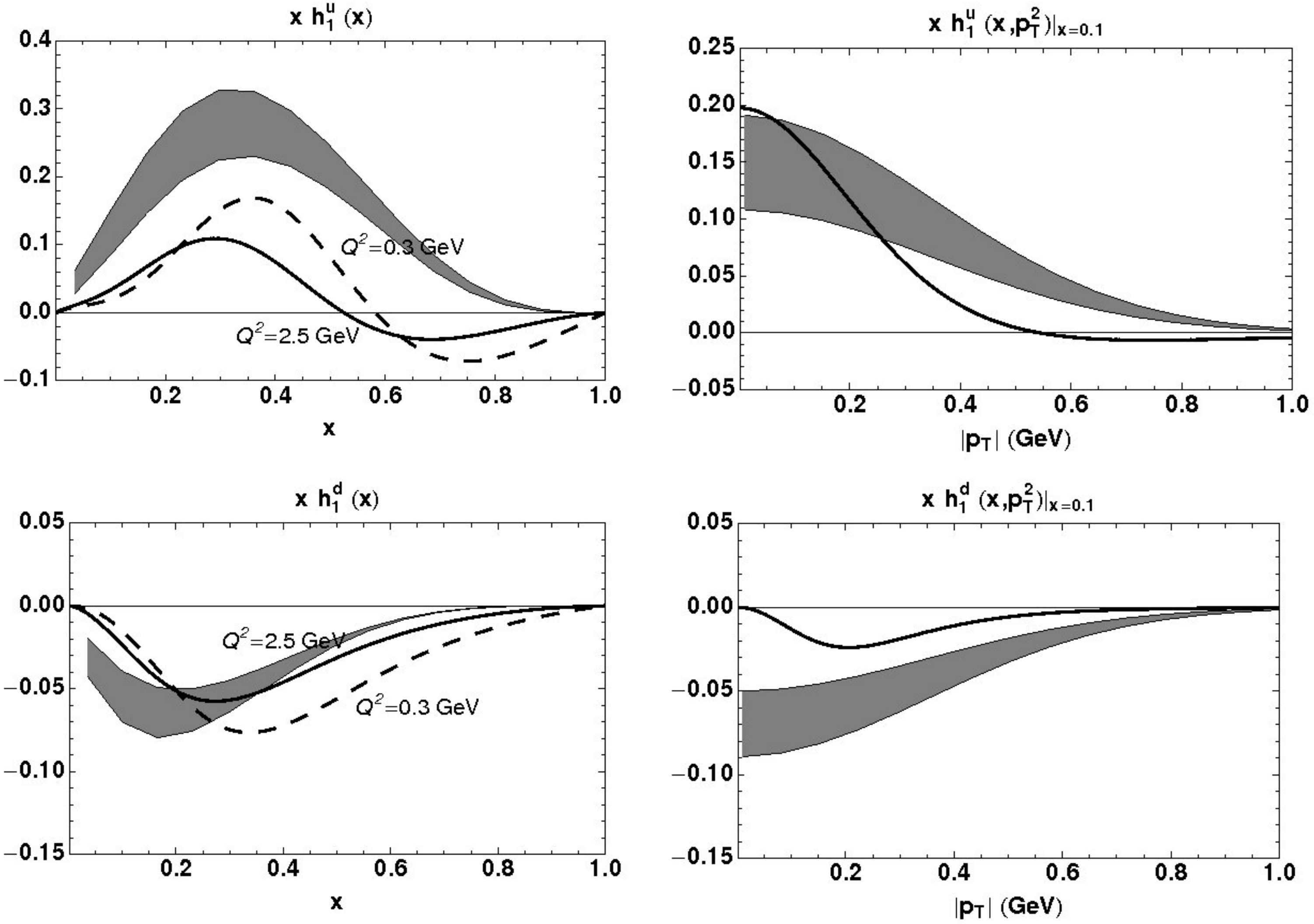
$$\boxed{g_{1L}(x,\mathbf{p}_T^2)-h_1(x,\mathbf{p}_T^2)\neq\frac{\mathbf{p}_T^2}{2M^2}\,h_{1T}^{\perp}(x,\mathbf{p}_T^2)}$$

Pretzelosity
Avakian *et al.*
P.R.D**78**, 114024 (08)

$$\boxed{h_1^{s}(x)=\frac{1}{2}\,\left[f_1^{s}(x)+g_1^{s}(x)\right]}$$

spin=0 Dq
saturates Soffer bound

transversity



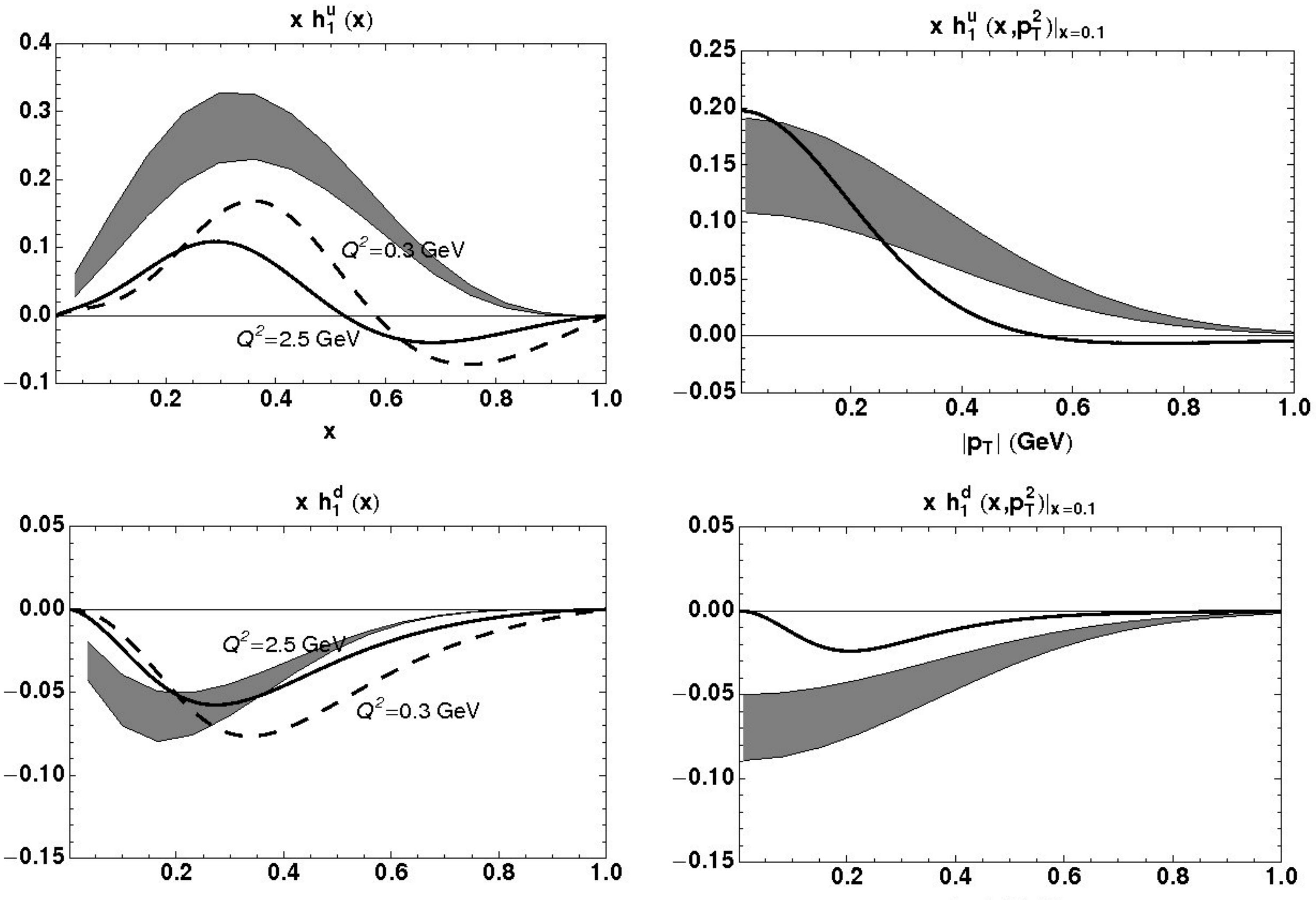
parametrization   Prokudin DIS08 arXiv:0807.0173 [hep-ph]

flavor-indep. $\mathbf{p}_T$ dependence ~ exp[-$\mathbf{p}_T^2$/<$\mathbf{p}_T^2$>]

factorized x dependence ~ $x^\alpha (1-x)^\beta$ no sign change allowed

evolution using code from
Hirai, Kumano, Miyama,
C.P.C.**111,**150(98)

transversity

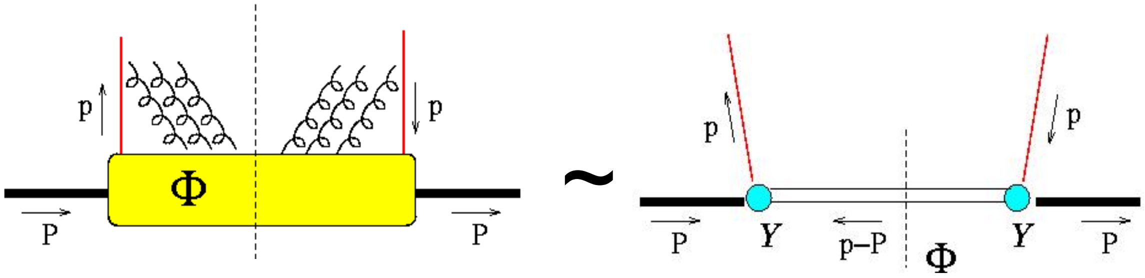


parametrization Prokudin DIS08 arXiv:0807.0173 [hep-ph]
flavor-indep. $\mathbf{p}_T$ dependence ~ $\exp[-\mathbf{p}_T^2/<\mathbf{p}_T^2>]$
factorized x dependence ~ $x^\alpha\ (1-x)^\beta$ no sign change allowed

evolution using code from Hirai, Kumano, Miyama, C.P.C.**111,**150(98)

diquark model : T-odd **TMD**

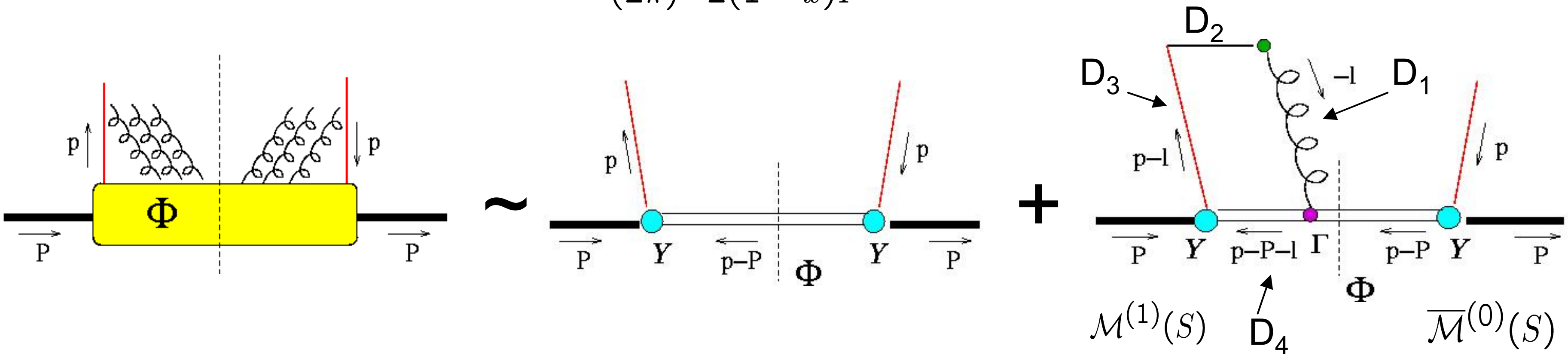
$$\Phi(x,\mathbf{p}_T,S) = \int \frac{d^4\xi}{(2\pi)^3}\, e^{ip\cdot\xi}\,\langle P,S|\bar{\psi}(0)\,U_{[0,\xi]}\,\psi(\xi)|P,S\rangle \;\approx\; \frac{1}{(2\pi)^3}\,\frac{1}{2(1-x)P^+}\,\overline{\mathcal{M}}^{(0)}(S)\,\mathcal{M}^{(0)}(S) +$$



diquark model : T-odd TMD

$$\Phi(x, \mathbf{p}_T, S) = \int \frac{d^4\xi}{(2\pi)^3} e^{ip\cdot\xi} \langle P, S|\bar{\psi}(0)\, U_{[0,\xi]}\, \psi(\xi)|P, S\rangle \approx \frac{1}{(2\pi)^3}\frac{1}{2(1-x)P^+}\overline{\mathcal{M}}^{(0)}(S)\,\mathcal{M}^{(0)}(S) +$$

$$\frac{1}{(2\pi)^3}\frac{1}{2(1-x)P^+}\left[\overline{\mathcal{M}}^{(0)}(S)\,\mathcal{M}^{(1)}(S) + \overline{\mathcal{M}}^{(1)}(S)\,\mathcal{M}^{(0)}(S)\right]$$

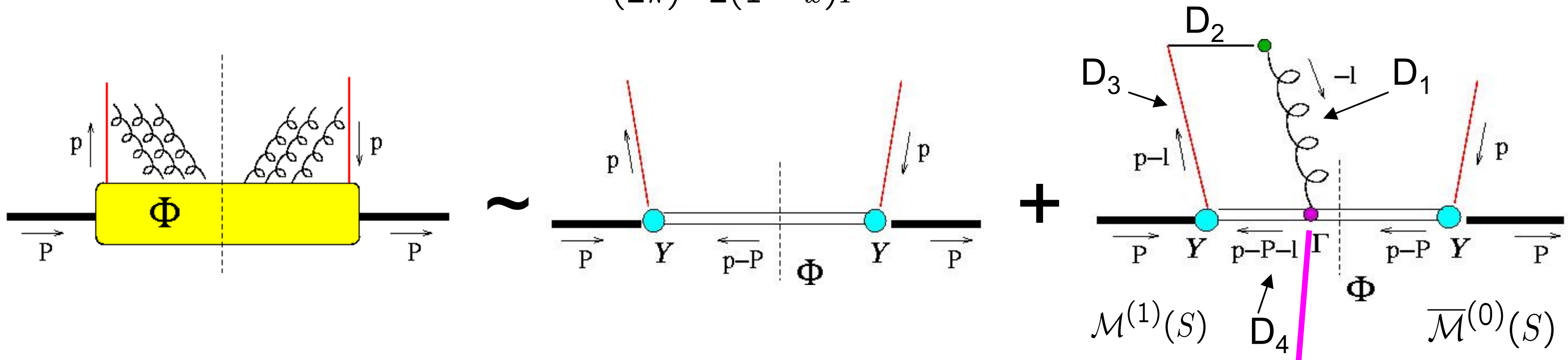


$$\mathcal{M}^{(1)}(S) = \int \frac{d^4l}{(2\pi)^4}\frac{i e\, n_-^\rho\, (\not{p} - \not{l} + m)}{D_1\, D_2\, D_3\, D_4}\left\{\begin{array}{c} \Gamma_{s\,\rho}\, \mathcal{Y}_s\, U(P,S) \\ \epsilon^*_\sigma(p-P,\lambda_a)\, \Gamma^{\nu\sigma}_{a\,\rho}\, d_{\mu\nu}(p-l-P)\, \mathcal{Y}^\mu_a\, U(P,S) \end{array}\right.$$

diquark model : T-odd TMD

$$\Phi(x,\mathbf{p}_T,S) = \int \frac{d^4\xi}{(2\pi)^3}\, e^{ip\cdot\xi}\, \langle P,S|\bar\psi(0)\, U_{[0,\xi]}\, \psi(\xi)|P,S\rangle \;\approx\; \frac{1}{(2\pi)^3}\,\frac{1}{2(1-x)P^+}\,\overline{\mathcal{M}}^{(0)}(S)\,\mathcal{M}^{(0)}(S) \,+$$

$$\frac{1}{(2\pi)^3}\,\frac{1}{2(1-x)P^+}\,\left[\overline{\mathcal{M}}^{(0)}(S)\,\mathcal{M}^{(1)}(S) \;+\; \overline{\mathcal{M}}^{(1)}(S)\,\mathcal{M}^{(0)}(S)\right]$$



$$\mathcal{M}^{(1)}(S) = \int \frac{d^4 l}{(2\pi)^4}\, \frac{ie\, n_-^{\rho}\, (\not p - \not l + m)}{D_1\, D_2\, D_3\, D_4} \left\{ \begin{array}{r} \boxed{\Gamma_{s\,\rho}}\; \mathcal{Y}_s\; U(P,S) \\ \epsilon^*_\sigma(p-P,\lambda_a)\, \boxed{\Gamma^{\nu\sigma}_{a\,\rho}}\; d_{\mu\nu}(p-l-P)\; \mathcal{Y}^{\mu}_{a}\; U(P,S) \end{array} \right.$$

$$\Gamma_{s\,\rho} \;=\; ie\,(2P-2p+l)_\rho$$

$$\Gamma^{\nu\sigma}_{a\,\rho} \;=\; -ie\,\left[(2P-2p+l)_\rho\, g^{\nu\sigma} \;-\; (P-p+(1+v)\,l)^\sigma\, g^{\nu}_{\rho} \;-\; (P-p-vl)^\nu\, g^{\sigma}_{\rho}\right] \xrightarrow{v\,\to\,1}$$

pointlike γWW
Peskin-Schröder

similar to
Robinson, Rizzo,
P.R.D**33** (86) 2608

v spin=1 Dq anom. magn. mom.

Cutkosky rules → put on-shell D_2, D_4 → analytic results (do not depend from v, but for other $d^{\mu\nu}$ they do)

our model cont'ed: T-odd **TMD**

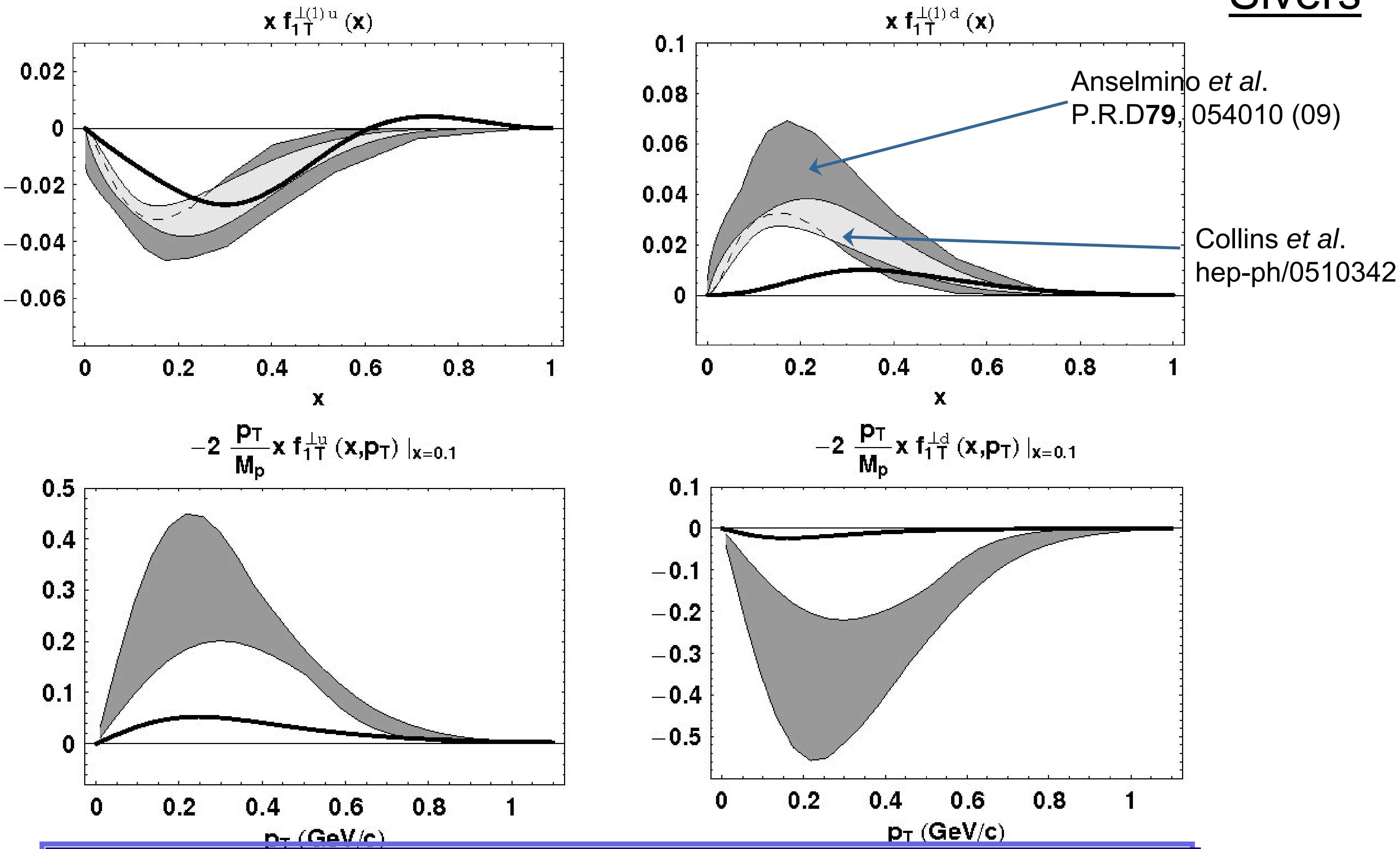
Sivers

$$
\begin{aligned}
\frac{\epsilon_T^{ij}\, p_{Ti}\, \hat{s}_{Tj}}{M}\, f_{1T}^{\perp}(x,\mathbf{p}_T^2) &= -\frac{1}{4}\,\frac{1}{(2\pi)^3}\,\frac{1}{2(1-x)P^+}\,\mathrm{Tr}\left[\left(\mathcal{M}^{(1)}(S)\,\overline{\mathcal{M}}^{(0)}(S) - \mathcal{M}^{(1)}(-S)\,\overline{\mathcal{M}}^{(0)}(-S)\right)\,\gamma^+\right] + \text{h.c.} \\
f_{1T}^{\perp\, s}(x,\mathbf{p}_T^2) &= -\frac{N_s^2}{(2\pi)^4}\,\frac{M\, e^2\,(m+xM)\,(1-x)^3}{4\, L_s^2(\Lambda_s^2)\,[\mathbf{p}_T^2 + L_s^2(\Lambda_s^2)]^3} \\
f_{1T}^{\perp\, a}(x,\mathbf{p}_T^2) &= \frac{N_a^2}{(2\pi)^4}\,\frac{M\, e^2\,(m+xM)\,x\,(1-x)^2}{4\, L_a^2(\Lambda_a^2)\,[\mathbf{p}_T^2 + L_a^2(\Lambda_a^2)]^3}
\end{aligned}
$$

Boer-Mulders

$$
\begin{aligned}
\frac{\epsilon_T^{ij}\, p_{Tj}}{M}\, h_1^{\perp}(x,\mathbf{p}_T^2) &= \frac{1}{4}\,\frac{1}{(2\pi)^3}\,\frac{1}{2(1-x)P^+}\,\mathrm{Tr}\left[\left(\mathcal{M}^{(1)}(S)\,\overline{\mathcal{M}}^{(0)}(S) + \mathcal{M}^{(1)}(-S)\,\overline{\mathcal{M}}^{(0)}(-S)\right)\, i\sigma^{i+}\gamma_5\right] + \text{h.c.} \\
h_1^{\perp\, s}(x,\mathbf{p}_T^2) &= f_{1T}^{\perp\, s}(x,\mathbf{p}_T^2) \\
h_1^{\perp\, a}(x,\mathbf{p}_T^2) &= -\frac{1}{x}\, f_{1T}^{\perp\, a}(x,\mathbf{p}_T^2)
\end{aligned}
$$

Sivers

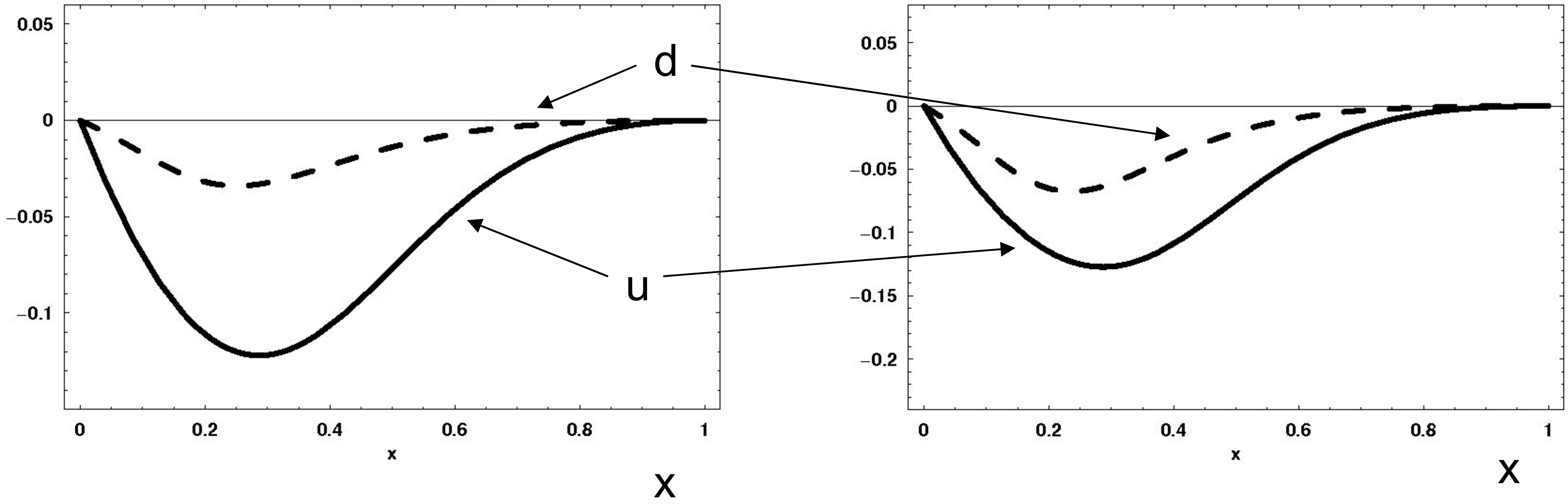


Anselmino *et al*.
P.R.D**79**, 054010 (09)

Collins *et al*.
hep-ph/0510342

parametrization: flavor-indep. $\mathbf{p}_T$ dependence ~ $\exp[-\mathbf{p}_T^2/\langle \mathbf{p}_T^2\rangle]$
factorized x dependence ~ $x^\alpha\,(1-x)^\beta$ no sign change allowed

Boer-Mulders moments



$x\, h_1^{\perp\,(1)}(x)$

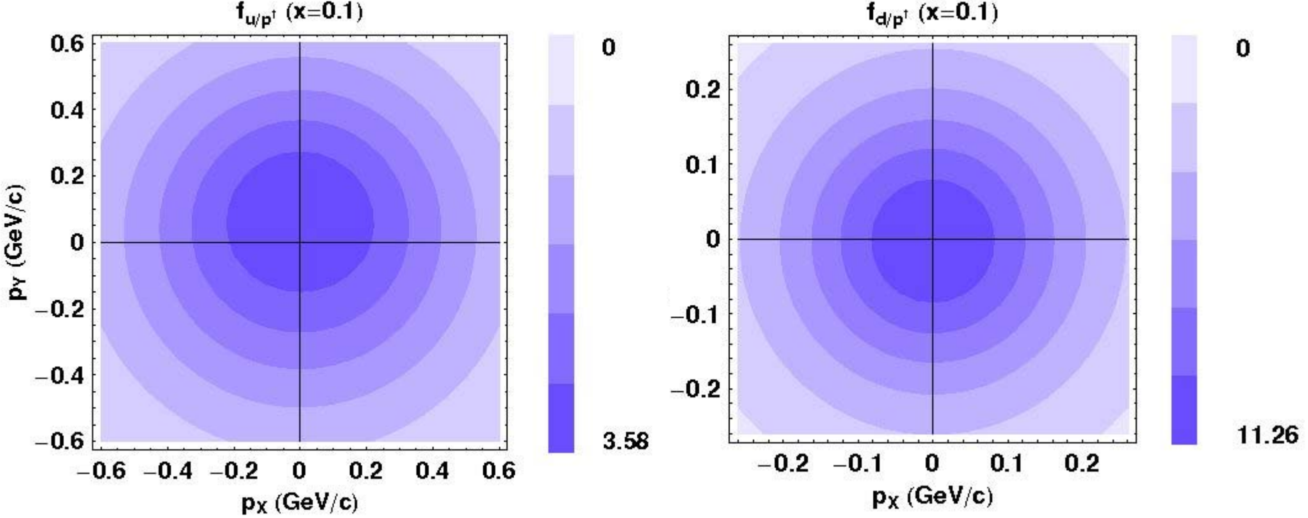
$x\, h_1^{\perp\,(1/2)}(x)$

agreement with sign of lattice calculations Hägler *et al*. (LHPC), P.R.D**77**, 094502 (08)

spin densities in $\mathbf{p}_T$ space

density of unpol. quarks q in $p^\uparrow$

$$f_{q/p^\uparrow} = f_1^q(x,\mathrm{p}_T^2) - f_{1T}^{\perp q}(x,\mathrm{p}_T^2)\,\frac{(\mathrm{p}_T\times\hat{\mathrm{s}}_T)\cdot\hat{\mathrm{P}}}{M}$$



Trento conventions for SIDIS

- **P** || z || **q**

if $s_{Tx} \Rightarrow$ deformation Δ

$\Delta \propto -\, p_{Ty} / M$

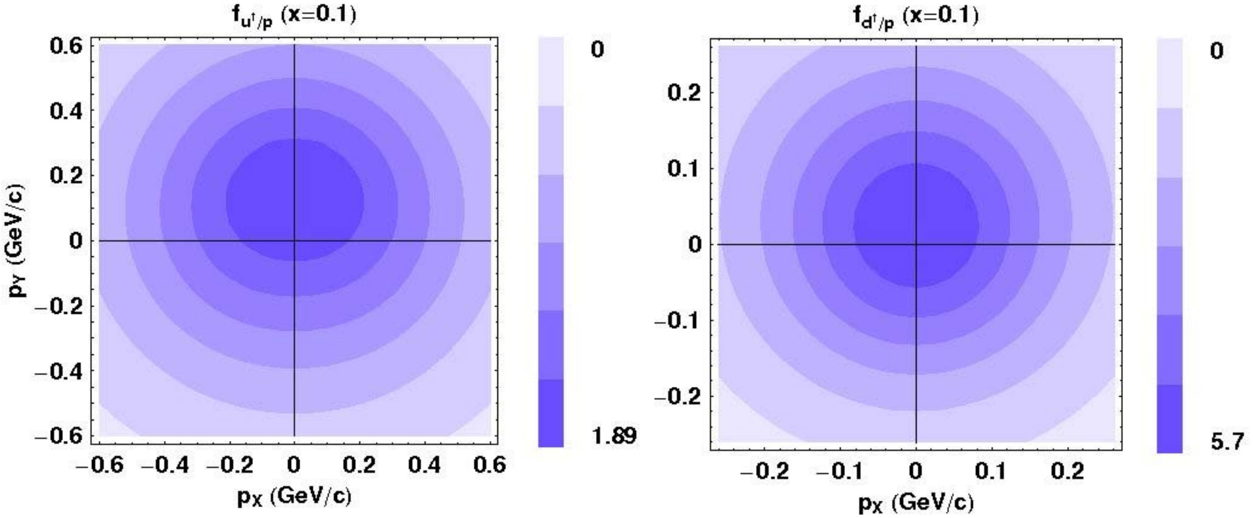
$f_{1T}^{\perp u} < 0 \;\Rightarrow \Delta > 0$

$f_{1T}^{\perp d} > 0 \;\Rightarrow \Delta < 0$

similarity with lattice
Hägler et al. (LHPC)
P.R.D**77**, 094502 (08)
but no direct comparison!

$$f_{q^\uparrow/p} = \frac{1}{2}\left[f_1^q(x,\mathrm{p}_T^2) - h_1^{\perp q}(x,\mathrm{p}_T^2)\,\frac{(\mathrm{p}_T\times\hat{\mathrm{s}}_{qT})\cdot\hat{\mathrm{P}}}{M}\right]$$

density of pol. quarks $q^\uparrow$ in unpol. p



if $s_{qTx} \Rightarrow$ deformation Δ

$\Delta \propto -\, p_{Ty} / M$

$h_1^{\perp u} < 0 \;\Rightarrow \Delta > 0$

$h_1^{\perp d} < 0 \;\Rightarrow \Delta > 0$

our model cont'ed: T-odd TMD as LCWF overlaps

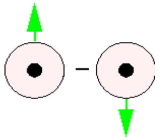
so far for Sivers fnct with spin=0 diquarks only

Brodsky, Gardner, P.L.**B643** (06) 22

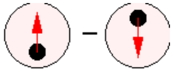
Zu, Schmidt, P.R.D**75** (07) 073008

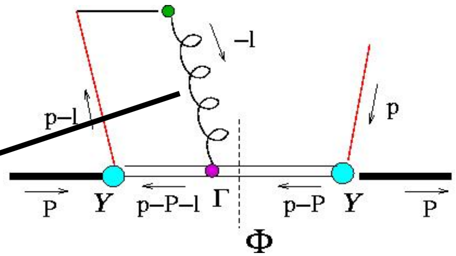
$\hat{\mathrm{P}} \parallel -\hat{z}$

$$\frac{2}{M}(\mathbf{p}_T \times \hat{\mathbf{s}}_T)\cdot\hat{z}\, f_{1T}^{\perp}(x,\mathbf{p}_T^2) = f_{q/p^\uparrow} - f_{q/p^\downarrow}$$



$$\frac{1}{M}(\mathbf{p}_T \times \hat{\mathbf{s}}_{qT})\cdot\hat{z}\, h_1^{\perp}(x,\mathbf{p}_T^2) = f_{q^\uparrow/p} - f_{q^\downarrow/p}$$





$$\frac{2}{M}(\mathbf{p}_T \times \hat{\mathbf{s}}_T)\cdot\hat{z}\, f_{1T}^{\perp\, s}(x,\mathbf{p}_T^2) = \int \frac{d\mathbf{p}'_T}{16\pi^3} \boxed{G(x,\mathbf{p}_T,\mathbf{p}'_T)} \sum_{\lambda_q} \left[\psi_{\lambda_q}^{\uparrow\,*}(x,\mathbf{p}_T)\,\psi_{\lambda_q}^{\uparrow}(x,\mathbf{p}'_T) - \psi_{\lambda_q}^{\downarrow\,*}(x,\mathbf{p}_T)\,\psi_{\lambda_q}^{\downarrow}(x,\mathbf{p}'_T)\right]$$

$$\frac{2}{M}(\mathbf{p}_T \times \hat{\mathbf{s}}_T)\cdot\hat{z}\, f_{1T}^{\perp\, a}(x,\mathbf{p}_T^2) = \int \frac{d\mathbf{p}'_T}{16\pi^3} \boxed{G(x,\mathbf{p}_T,\mathbf{p}'_T)} \sum_{\lambda_q,\,\lambda_a} \left[\psi_{\lambda_q,\,\lambda_a}^{\uparrow\,*}(x,\mathbf{p}_T)\,\psi_{\lambda_q,\,\lambda_a}^{\uparrow}(x,\mathbf{p}'_T) - \psi_{\lambda_q,\,\lambda_a}^{\downarrow\,*}(x,\mathbf{p}_T)\,\psi_{\lambda_q,\,\lambda_a}^{\downarrow}(x,\mathbf{p}'_T)\right]$$

$$\frac{1}{M}(\mathbf{p}_T \times \hat{\mathbf{s}}_{qT})\cdot\hat{z}\, h_1^{\perp\, s}(x,\mathbf{p}_T^2) = \int \frac{d\mathbf{p}'_T}{16\pi^3} \boxed{G(x,\mathbf{p}_T,\mathbf{p}'_T)} \frac{1}{2}\sum_{\lambda_N} \left[\psi_{\uparrow\,*}^{\lambda_N}(x,\mathbf{p}_T)\,\psi_{\uparrow}^{\lambda_N}(x,\mathbf{p}'_T) - \psi_{\downarrow\,*}^{\lambda_N}(x,\mathbf{p}_T)\,\psi_{\downarrow}^{\lambda_N}(x,\mathbf{p}'_T)\right]$$

$$\frac{1}{M}(\mathbf{p}_T \times \hat{\mathbf{s}}_{qT})\cdot\hat{z}\, h_1^{\perp\, a}(x,\mathbf{p}_T^2) = \int \frac{d\mathbf{p}'_T}{16\pi^3} \boxed{G(x,\mathbf{p}_T,\mathbf{p}'_T)} \frac{1}{2}\sum_{\lambda_N,\,\lambda_a} \left[\psi_{\uparrow,\,\lambda_a}^{\lambda_N\,*}(x,\mathbf{p}_T)\,\psi_{\uparrow,\,\lambda_a}^{\lambda_N}(x,\mathbf{p}'_T) - \psi_{\downarrow,\,\lambda_a}^{\lambda_N\,*}(x,\mathbf{p}_T)\,\psi_{\downarrow,\,\lambda_a}^{\lambda_N}(x,\mathbf{p}'_T)\right]$$

FSI operator G universal

$$\mathrm{Im}\, G(x,\mathbf{p}_T,\mathbf{p}'_T) = -\frac{e^2}{2\,(2\pi)^2}\,\frac{1}{(\mathbf{p}_T - \mathbf{p}'_T)^2}$$

Sivers function and anomalous magnetic moment

N anom. magn. mom. κ also described via overlap of lcwf   Zu, Schmidt, P.R.D**75** (07) 073008

$$\mathbf{p}'_T = \mathbf{p}_T + (1-x)\,\mathbf{q}_T$$

$$\kappa_s = \lim_{q\to 0} \frac{e}{2M} F_2(q) = -\frac{1}{q_x - i q_y} \int \frac{d\mathbf{p}_T\, dx}{16\pi^3} \sum_{\lambda_q} \left[\psi^{+\,*}_{\lambda_q}(x,\mathbf{p}'_T)\, \psi^{-}_{\lambda_q}(x,\mathbf{p}_T) \right] \Big|_{\mathbf{p}'_T=\mathbf{p}_T} = \int_0^1 dx\ \kappa_s(x)$$

$$\kappa_a = \int_0^1 dx\ \kappa_a(x)$$

$$\kappa_s(x) = \frac{N_s^2}{(2\pi)^2}\, \frac{1-x}{12}\, \frac{(1-x)^3\,(m+xM)}{[L_s^2(\Lambda_s^2)]^3}$$

$$\kappa_a(x) = -\frac{N_a^2}{(2\pi)^2}\, \frac{x}{12}\, \frac{(1-x)^3\,(m+xM)}{[L_a^2(\Lambda_a^2)]^3}$$

from model input calculate

$$f_{1T}^{\perp s}(x) = \int d\mathbf{p}_T\ f_{1T}^{\perp s}(x,\mathbf{p}_T^2) = -\frac{3M\,e^2}{8\pi}\, \frac{\kappa_s(x)}{1-x}$$

$$f_{1T}^{\perp a}(x) = \int d\mathbf{p}_T\ f_{1T}^{\perp a}(x,\mathbf{p}_T^2) = -\frac{3M\,e^2}{8\pi}\, \frac{\kappa_a(x)}{1-x}$$

$$\boxed{\begin{aligned} \int_0^1 dx\ (1-x)\ f_{1T}^{\perp s}(x) &= -\frac{3M\,e^2}{8\pi}\,\kappa_s \\ \int_0^1 dx\ (1-x)\ f_{1T}^{\perp a}(x) &= -\frac{3M\,e^2}{8\pi}\,\kappa_a \end{aligned}}$$

isospin symmetry + valence picture N={qq q'}

$$\left.\begin{aligned} \kappa_p &= 2\,e_u\,\kappa_u + e_d\,\kappa_d = 1.79 \\ \kappa_n &= 2\,e_d\,\kappa_u + e_u\,\kappa_d = -\,1.91 \end{aligned}\right\} \longrightarrow \left\{\begin{aligned} \kappa_u &= 0.835 \\ \kappa_d &= -\,2.03 \end{aligned}\right.$$

spectator diquark N={q Dq}

$$\kappa_u = c_s^2\,\kappa_s + c_a^2\,\kappa_a = 0.997$$

$$\kappa_d = c_{a'}^2\,\kappa_{a'} = -\,0.345$$

Results: weighted SSA

why?

Results: weighted SSA

why?

unweighted SSA $\Rightarrow$ $\dfrac{d\sigma}{.....d\mathbf{P}_{h\perp}}$ $\Rightarrow$ $\mathcal{C}\ [\ w\ f_1\ f_2\ ]$

Ex: SIDIS

$$\mathcal{C}[w\, f\, D] = x \sum_q e_q^2 \int d^2\mathbf{p}_T d^2\mathbf{k}_T\, \delta^{(2)}(\mathbf{p}_T - \mathbf{k}_T - \mathbf{P}_{h\perp}/z)\, w(\mathbf{p}_T, \mathbf{k}_T)\, f^q(x, \mathbf{p}_T^2)\, D^q(z, \mathbf{k}_T^2)$$

need numerical calculations (next step of our work; work is in progress..) or break convolution with model $\mathbf{p}_T$ and $\mathbf{k}_T$ dependence in f^q and D^q

→ gaussian dependence not consistent with our model $\mathbf{p}_T$ dependence

Results: weighted SSA

why?

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need numerical calculations (next step of our work; work is in progress..) or break convolution with model $\mathbf{p}_T$ and $\mathbf{k}_T$ dependence in f^q and D^q

→ gaussian dependence $\boxed{\text{not consistent with our model } \mathbf{p}_T \text{ dependence}}$

Results: weighted SSA

why?

unweighted SSA → $\frac{d\sigma}{.....d\mathbf{P}_{h\perp}}$ → $\mathcal{C}\ [\ w\ f_1\ f_2\ ]$

Ex: SIDIS

$$\mathcal{C}[w\,f\,D] = x\sum_q e_q^2 \int d^2\mathbf{p}_T d^2\mathbf{k}_T\, \delta^{(2)}(\mathbf{p}_T - \mathbf{k}_T - \mathbf{P}_{h\perp}/z)\, w(\mathbf{p}_T,\mathbf{k}_T)\, f^q(x,\mathbf{p}_T^2)\, D^q(z,\mathbf{k}_T^2)$$

need numerical calculations (next step of our work; work is in progress..) or break convolution with model $\mathbf{p}_T$ and $\mathbf{k}_T$ dependence in f^q and D^q

→ gaussian dependence   not consistent with our model $\mathbf{p}_T$ dependence

SIDIS SSA prototype

$$A(x,z,Q^2) \propto \frac{\sum_q e_q^2\ \ f^q\ \ D^q}{\sum_q e_q^2\ \ x f_1^q(x,Q^2)\ \ D_1^q(z,Q^2)}$$

If $f=f_1,g_1,h_1$
evolution→ Q^2
Miyama, Kumano, C.P.C. **94**,185(96)
or Hirai *et al*., C.P.C. **111**, 150 (98)

If $D=D_1$
evolution → DSS
P.R.D**75**,114010(07)
fav.+ unfav.

Drell-Yan SSA prototype

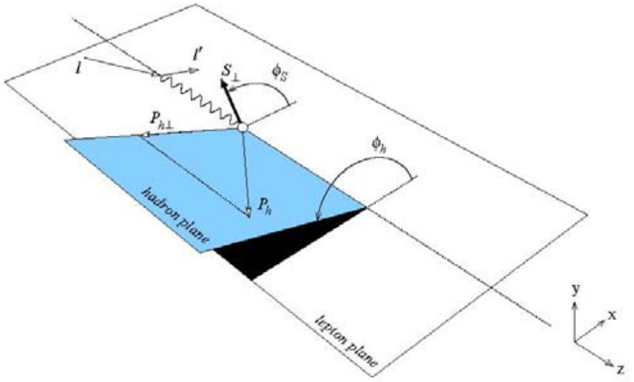
$$A(x_1,x_2,Q^2) \propto \frac{\sum_q e_q^2\ \ f_A^{\bar q}\ \ f_B^q}{\sum_q e_q^2\ \ x_1 f_1^{\bar q}(x_1,Q^2)\ \ x_2 f_1^q(x_2,Q^2)}$$

If f_A or f_B = f_1,g_1,h_1
evolution→ Q^2 ditto

valence approximation

weighted SSA : SIDIS

$$A_{XY}^{W}(x,y,z) \;\propto\; \frac{\langle W\rangle_{XY}}{\langle 1\rangle_{UU}} \equiv \frac{\int d\phi_S d\phi_h d^2\mathbf{P}_{h\perp}\, W\, d\sigma_{XY}}{\int d\phi_S d\phi_h d^2\mathbf{P}_{h\perp}\, d\sigma_{UU}}$$



$$A_{UU}^{\frac{Q_T^2}{4MM_h}\cos 2\phi_h} = 2\,\frac{\langle\frac{Q_T^2}{4MM_h}\cos 2\phi_h\rangle_{UU}}{\langle 1\rangle_{UU}} = 2\,\frac{B(y)}{A(y)}\,\frac{\sum_q e_q^2\, x h_1^{\perp(1)q}(x)\, H_1^{\perp(1)q}(z)}{\sum_q e_q^2\, x f_1^q(x)\, D_1^q(z)}$$

$$A_{UL}^{\frac{Q_T^2}{4MM_h}\sin 2\phi_h} = 2\,\frac{\langle\frac{Q_T^2}{4MM_h}\sin 2\phi_h\rangle_{UL}}{\langle 1\rangle_{UU}} = 2\,\frac{B(y)}{A(y)}\,\frac{\sum_q e_q^2\, x h_{1L}^{\perp(1)q}(x)\, H_1^{\perp(1)q}(z)}{\sum_q e_q^2\, x f_1^q(x)\, D_1^q(z)}$$

$$A_{UT}^{\frac{Q_T}{M_h}\sin(\phi_h+\phi_S)} = 2\,\frac{\langle\frac{Q_T}{M_h}\sin(\phi_h+\phi_S)\rangle_{UT}}{\langle 1\rangle_{UU}} = 2\,\frac{B(y)}{A(y)}\,\frac{\sum_q e_q^2\, x h_1^q(x)\, H_1^{\perp(1)q}(z)}{\sum_q e_q^2\, x f_1^q(x)\, D_1^q(z)}$$

$$A_{UT}^{\frac{Q_T}{M}\sin(\phi_h-\phi_S)} = 2\,\frac{\langle\frac{Q_T}{M}\sin(\phi_h-\phi_S)\rangle_{UT}}{\langle 1\rangle_{UU}} = -2\,\frac{A(y)}{A(y)}\,\frac{\sum_q e_q^2\, x f_{1T}^{\perp(1)q}(x)\, D_1^q(z)}{\sum_q e_q^2\, x f_1^q(x)\, D_1^q(z)}$$

$$A_{UT}^{\frac{Q_T^3}{6M^2M_h}\sin(3\phi_h-\phi_S)} = 2\,\frac{\langle\frac{Q_T^3}{6M^2M_h}\sin(3\phi_h-\phi_S)\rangle_{UT}}{\langle 1\rangle_{UU}} = 2\,\frac{B(y)}{A(y)}\,\frac{\sum_q e_q^2\, x h_{1T}^{\perp(2)q}(x)\, H_1^{\perp(1)q}(z)}{\sum_q e_q^2\, x f_1^q(x)\, D_1^q(z)}$$

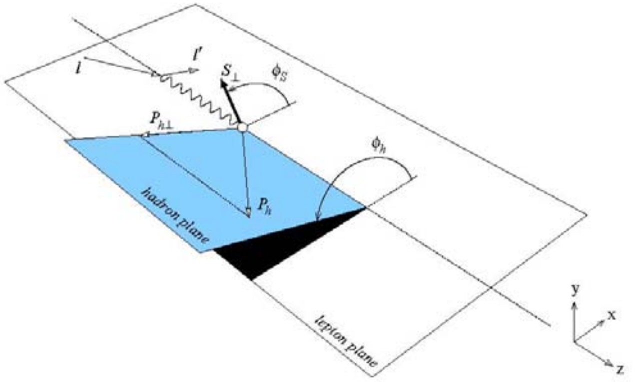
$$A_{LT}^{\frac{Q_T}{M}\cos(\phi_h-\phi_S)} = 2\,\frac{\langle\frac{Q_T}{M}\cos(\phi_h-\phi_S)\rangle_{UT}}{\langle 1\rangle_{UU}} = 2\,\frac{C(y)}{A(y)}\,\frac{\sum_q e_q^2\, x g_{1T}^{(1)q}(x)\, D_1^q(z)}{\sum_q e_q^2\, x f_1^q(x)\, D_1^q(z)}$$

$$\begin{aligned} A(y) &= \frac{1-y+\frac{1}{2}y^2+\frac{1}{4}y^2\gamma^2}{x^2y^2\,(1+\gamma^2)} \\ B(y) &= \frac{1-y-\frac{1}{4}y^2\gamma^2}{x^2y^2\,(1+\gamma^2)} \\ C(y) &= \frac{y(1-\frac{1}{2}y)}{x^2y^2\,\sqrt{1+\gamma^2}} \\ \gamma(x) &= \frac{2Mx}{Q} \end{aligned}$$

$$f^{(n)}(x) = \int d^2\mathbf{p}_T \left(\frac{\mathbf{p}_T^2}{2M^2}\right)^{n} f(x,\mathbf{p}_T^2)$$

weighted SSA : SIDIS

$$A_{XY}^{W}(x,y,z) \;\propto\; \frac{\langle W\rangle_{XY}}{\langle 1\rangle_{UU}} \equiv \frac{\int d\phi_S d\phi_h d^2\mathbf{P}_{h\perp}\, W\, d\sigma_{XY}}{\int d\phi_S d\phi_h d^2\mathbf{P}_{h\perp}\, d\sigma_{UU}}$$



$$A_{UU}^{Q_T^2\,\cos 2\phi_h} = 2\,\frac{\langle \frac{\boxed{Q_T^2}}{4MM_h}\,\cos 2\phi_h\rangle_{UU}}{\langle 1\rangle_{UU}} = 2\,\frac{B(y)}{A(y)}\,\frac{\sum_q e_q^2\, x h_1^{\perp(1)q}(x)\, H_1^{\perp(1)q}(z)}{\sum_q e_q^2\, x f_1^q(x)\, D_1^q(z)}$$

but later…

$$A(y) = \frac{1-y+\frac{1}{2}y^2+\frac{1}{4}y^2\gamma^2}{x^2y^2\,(1+\gamma^2)}$$

$$B(y) = \frac{1-y-\frac{1}{4}y^2\gamma^2}{x^2y^2\,(1+\gamma^2)}$$

$$C(y) = \frac{y(1-\frac{1}{2}y)}{x^2y^2\,\sqrt{1+\gamma^2}}$$

$$\gamma(x) = \frac{2Mx}{Q}$$

$$A_{UL}^{Q_T^2\,\sin 2\phi_h} = 2\,\frac{\langle \frac{\boxed{Q_T^2}}{4MM_h}\,\sin 2\phi_h\rangle_{UL}}{\langle 1\rangle_{UU}} = 2\,\frac{B(y)}{A(y)}\,\frac{\sum_q e_q^2\, x h_{1L}^{\perp(1)q}(x)\, H_1^{\perp(1)q}(z)}{\sum_q e_q^2\, x f_1^q(x)\, D_1^q(z)}$$

$$A_{UT}^{Q_T\,\sin(\phi_h+\phi_S)} = 2\,\frac{\langle \frac{Q_T}{M_h}\,\sin(\phi_h+\phi_S)\rangle_{UT}}{\langle 1\rangle_{UU}} = 2\,\frac{B(y)}{A(y)}\,\frac{\sum_q e_q^2\, x h_1^q(x)\, H_1^{\perp(1)q}(z)}{\sum_q e_q^2\, x f_1^q(x)\, D_1^q(z)}$$

$$A_{UT}^{Q_T\,\sin(\phi_h-\phi_S)} = 2\,\frac{\langle \frac{Q_T}{M}\,\sin(\phi_h-\phi_S)\rangle_{UT}}{\langle 1\rangle_{UU}} = -2\,\frac{A(y)}{A(y)}\,\frac{\sum_q e_q^2\, x f_{1T}^{\perp(1)q}(x)\, D_1^q(z)}{\sum_q e_q^2\, x f_1^q(x)\, D_1^q(z)}$$

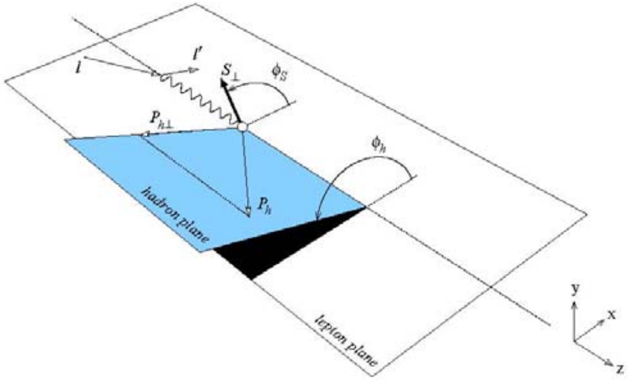
$$A_{UT}^{Q_T^3\,\sin(3\phi_h-\phi_S)} = 2\,\frac{\langle \frac{\boxed{Q_T^3}}{6M^2M_h}\,\sin(3\phi_h-\phi_S)\rangle_{UT}}{\langle 1\rangle_{UU}} = 2\,\frac{B(y)}{A(y)}\,\frac{\sum_q e_q^2\, x h_{1T}^{\perp(2)q}(x)\, H_1^{\perp(1)q}(z)}{\sum_q e_q^2\, x f_1^q(x)\, D_1^q(z)}$$

$$A_{LT}^{Q_T\,\cos(\phi_h-\phi_S)} = 2\,\frac{\langle \frac{Q_T}{M}\,\cos(\phi_h-\phi_S)\rangle_{UT}}{\langle 1\rangle_{UU}} = 2\,\frac{C(y)}{A(y)}\,\frac{\sum_q e_q^2\, x g_{1T}^{(1)q}(x)\, D_1^q(z)}{\sum_q e_q^2\, x f_1^q(x)\, D_1^q(z)}$$

$$f^{(n)}(x) = \int d^2\mathbf{p}_T \left(\frac{\mathbf{p}_T^2}{2M^2}\right)^{n} f(x,\mathbf{p}_T^2)$$

weighted SSA :   SIDIS

$$A_{XY}^{W}(x,y,z) \;\propto\; \frac{\langle W\rangle_{XY}}{\langle 1\rangle_{UU}} \equiv \frac{\int d\phi_S d\phi_h d^2\mathbf{P}_{h\perp}\, W\, d\sigma_{XY}}{\int d\phi_S d\phi_h d^2\mathbf{P}_{h\perp}\, d\sigma_{UU}}$$



$$A_{UU}^{Q_T^2 \cos 2\phi_h} = 2\,\frac{\langle \frac{Q_T^2}{4MM_h} \cos 2\phi_h\rangle_{UU}}{\langle 1\rangle_{UU}} = 2\,\frac{B(y)}{A(y)}\,\frac{\sum_q e_q^2\, x h_1^{\perp(1)q}(x)\, H_1^{\perp(1)q}(z)}{\sum_q e_q^2\, x f_1^q(x)\, D_1^q(z)}$$

but later…

$$A(y) = \frac{1-y+\frac{1}{2}y^2+\frac{1}{4}y^2\gamma^2}{x^2y^2\,(1+\gamma^2)}$$

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$$A_{UT}^{Q_T \sin(\phi_h+\phi_S)} = 2\,\frac{\langle \frac{Q_T}{M_h} \sin(\phi_h+\phi_S)\rangle_{UT}}{\langle 1\rangle_{UU}} = 2\,\frac{B(y)}{A(y)}\,\frac{\sum_q e_q^2\, x h_1^q(x)\, H_1^{\perp(1)q}(z)}{\sum_q e_q^2\, x f_1^q(x)\, D_1^q(z)}$$

$$A_{UT}^{Q_T \sin(\phi_h-\phi_S)} = 2\,\frac{\langle \frac{Q_T}{M} \sin(\phi_h-\phi_S)\rangle_{UT}}{\langle 1\rangle_{UU}} = -2\,\frac{A(y)}{A(y)}\,\frac{\sum_q e_q^2\, x f_{1T}^{\perp(1)q}(x)\, D_1^q(z)}{\sum_q e_q^2\, x f_1^q(x)\, D_1^q(z)}$$

HERMES
preliminary (old) data

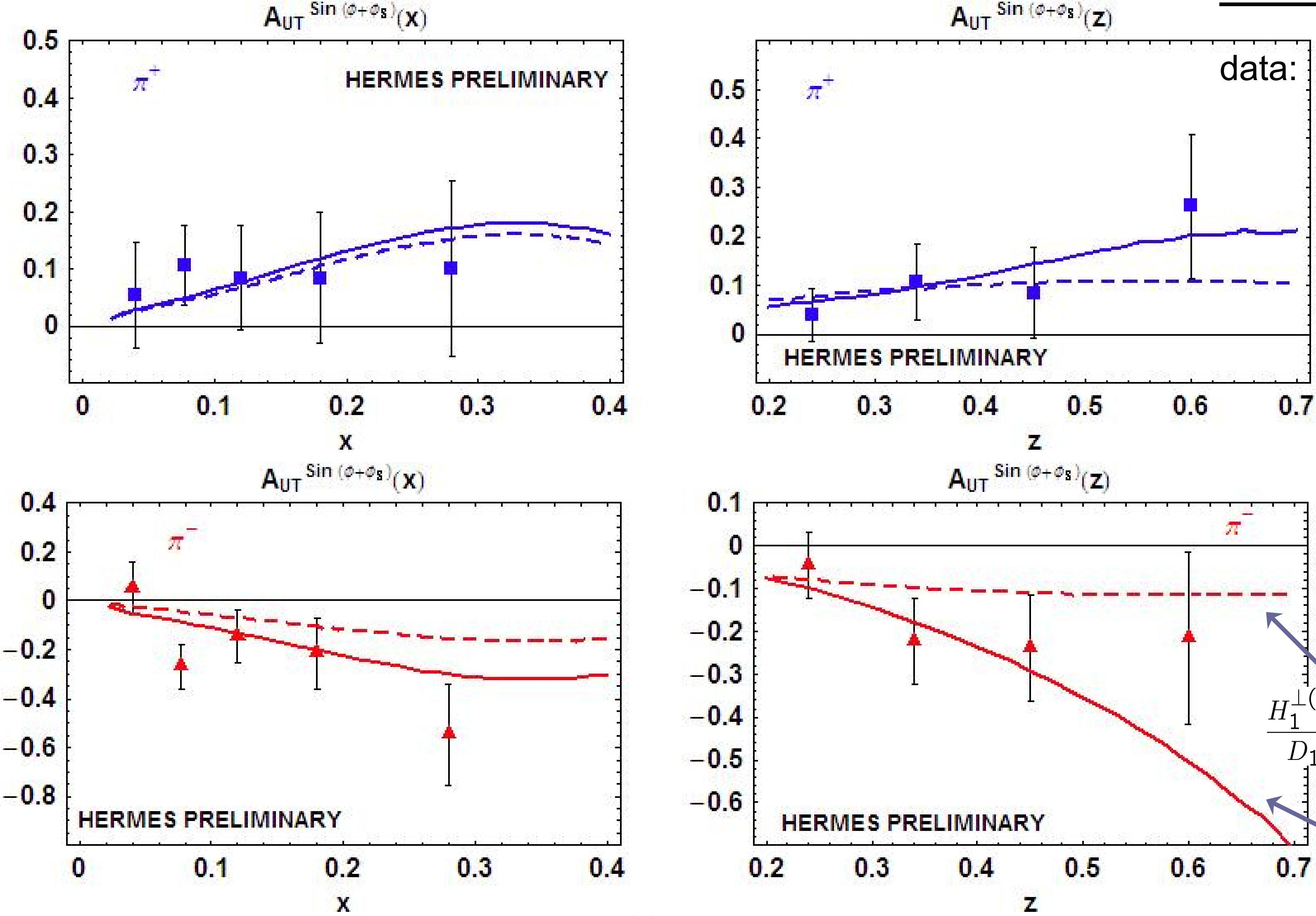
$$A_{UT}^{Q_T^3 \sin(3\phi_h-\phi_S)} = 2\,\frac{\langle \frac{Q_T^3}{6M^2M_h} \sin(3\phi_h-\phi_S)\rangle_{UT}}{\langle 1\rangle_{UU}} = 2\,\frac{B(y)}{A(y)}\,\frac{\sum_q e_q^2\, x h_{1T}^{\perp(2)q}(x)\, H_1^{\perp(1)q}(z)}{\sum_q e_q^2\, x f_1^q(x)\, D_1^q(z)}$$

$$A_{LT}^{Q_T \cos(\phi_h-\phi_S)} = 2\,\frac{\langle \frac{Q_T}{M} \cos(\phi_h-\phi_S)\rangle_{UT}}{\langle 1\rangle_{UU}} = 2\,\frac{C(y)}{A(y)}\,\frac{\sum_q e_q^2\, x g_{1T}^{(1)q}(x)\, D_1^q(z)}{\sum_q e_q^2\, x f_1^q(x)\, D_1^q(z)}$$

prediction

$$f^{(n)}(x) = \int d^2\mathbf{p}_T \left(\frac{\mathbf{p}_T^2}{2M^2}\right)^n f(x,\mathbf{p}_T^2)$$

Collins effect



data: Seidl (HERMES)
DIS04

s=56.2 GeV²
<Q²>=2.5 "
$y_{min}(x)<y<0.85$
0.023<x<0.4

$$\frac{H_1^{\perp(1)}(z)}{D_1(z)}\; D_1^{DSS}(z,Q^2)$$

$$H_1^{\perp(1)}(z)$$

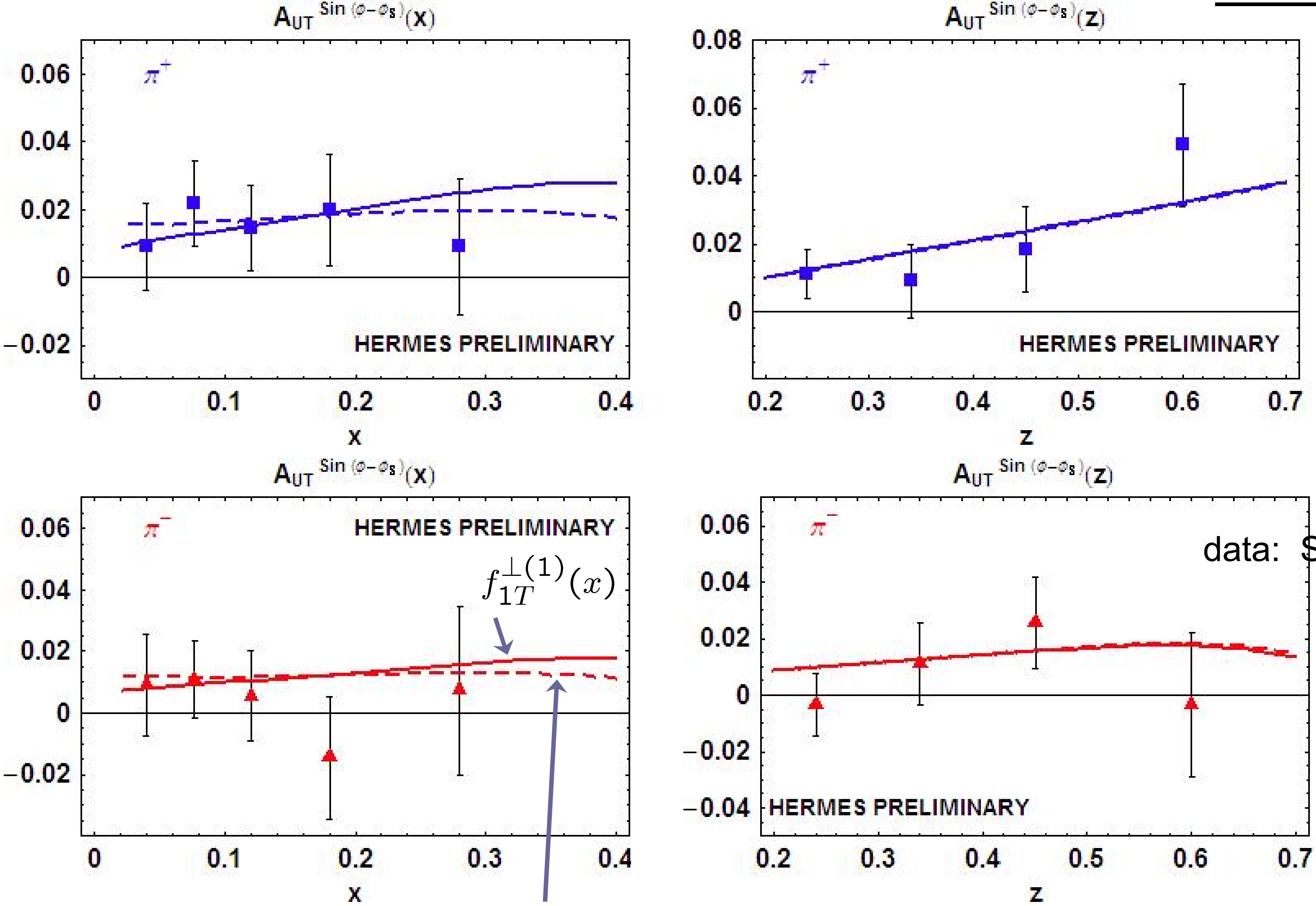
$$A_{UT}^{Q_T\,\sin(\phi_h+\phi_S)} = 2\,\frac{B(y)}{A(y)}\,\frac{\sum_q e_q^2\,x h_1^q(x)\,H_1^{\perp(1)q}(z)}{\sum_q e_q^2\,x f_1^q(x)\,D_1^q(z)}$$

spect. Dq model of

Bacchetta *et al.*, P.L.**B659**,234 (08)

$$H_1^{\perp fav} = -H_1^{\perp unfav}$$

Sivers effect



data: Seidl (HERMES

DIS04

s=56.2 GeV^2

<Q^2>=2.5 “

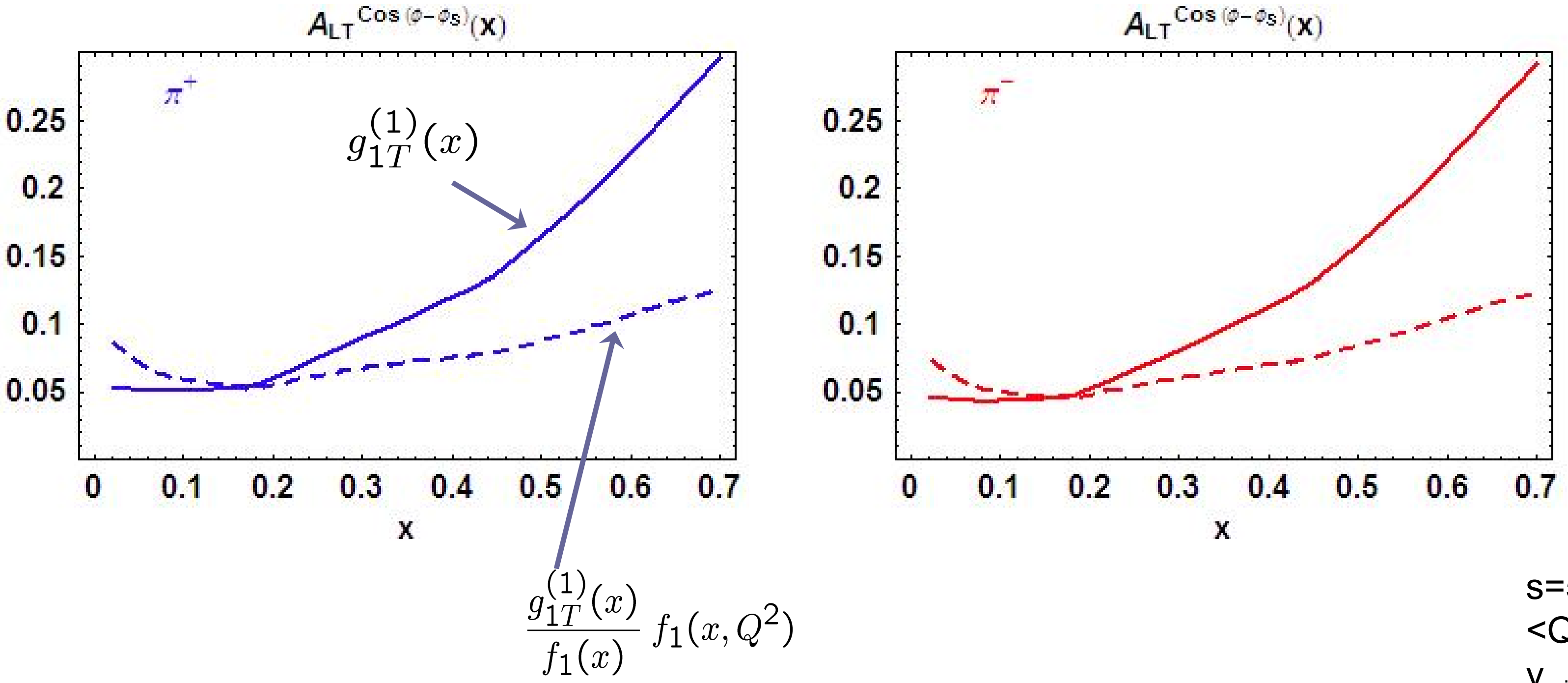
$y_{min}(x)<y<0.85$

$0.023<x<0.4$

$$\frac{f_{1T}^{\perp(1)}(x)}{f_1(x)}\, f_1(x,Q^2)$$

$$A_{UT}^{Q_T\,\sin(\phi_h-\phi_S)} = -2\,\frac{A(y)}{A(y)}\,\frac{\sum_q e_q^2\, x f_{1T}^{\perp(1)q}(x)\, D_1^q(z)}{\sum_q e_q^2\, x f_1^q(x)\, D_1^q(z)}$$

$A_{LT}^{\cos(\phi-\phi s)}$ in SIDIS



$g_{1T}^{(1)}(x)$

$\frac{g_{1T}^{(1)}(x)}{f_1(x)}\, f_1(x, Q^2)$

s=56.2 GeV^2
<Q^2>=2.5 “
y_{min}(x)<y<0.85
0.023<x<0.4

$$A_{LT}^{Q_T\,\cos(\phi_h-\phi_S)} \;=\; 2\,\frac{C(y)}{A(y)}\,\frac{\sum_q e_q^2\, x g_{1T}^{(1)q}(x)\, D_1^q(z)}{\sum_q e_q^2\, x f_1^q(x)\, D_1^q(z)}$$

$A_{UU}^{\cos 2\phi}$ in SIDIS

$$\frac{d\sigma^o}{dx\,dy\,dz\,d\phi_h\,d\mathbf{P}_{h\perp}} \propto F_{UU,T} + \epsilon\, F_{UU,L} + \sqrt{2\epsilon(1+\epsilon)}\, \cos\phi_h\, F_{UU}^{\cos\phi_h} + \epsilon \cos 2\phi_h\, F_{UU}^{\cos 2\phi_h}$$

$A_{UU}^{\cos 2\phi}$ in SIDIS

$$\frac{d\sigma^o}{dx\,dy\,dz\,d\phi_h\,d\mathbf{P}_{h\perp}} \propto F_{UU,T} + \epsilon\, F_{UU,L} + \sqrt{2\epsilon(1+\epsilon)}\,\cos\phi_h\, F_{UU}^{\cos\phi_h} + \epsilon\cos 2\phi_h\, F_{UU}^{\cos 2\phi_h}$$

Hp: $$F_{UU}^{\cos 2\phi} = F_{UU}^{\cos 2\phi}|_{pQCD} + F_{UU}^{\cos 2\phi}|_{BM}$$

high p_T low p_T

see, e.g., Bacchetta *et al*., JHEP **08**, 023 (08) $$F_{UU}^{\cos 2\phi}|_{pQCD} \approx \frac{1}{2} F_{UU,L}|_{pQCD}$$

$$\int d\phi_h\, d\sigma^o \propto F_{UU,T} + \epsilon\, F_{UU,L}$$ + Rosenbluth separation

$A_{UU}^{\cos 2\phi}$ in SIDIS

$$\frac{d\sigma^o}{dx\,dy\,dz\,d\phi_h\,d\mathbf{P}_{h\perp}} \propto F_{UU,T} + \epsilon\, F_{UU,L} + \sqrt{2\epsilon(1+\epsilon)}\,\cos\phi_h\, F_{UU}^{\cos\phi_h} + \epsilon\cos 2\phi_h\, F_{UU}^{\cos 2\phi_h}$$

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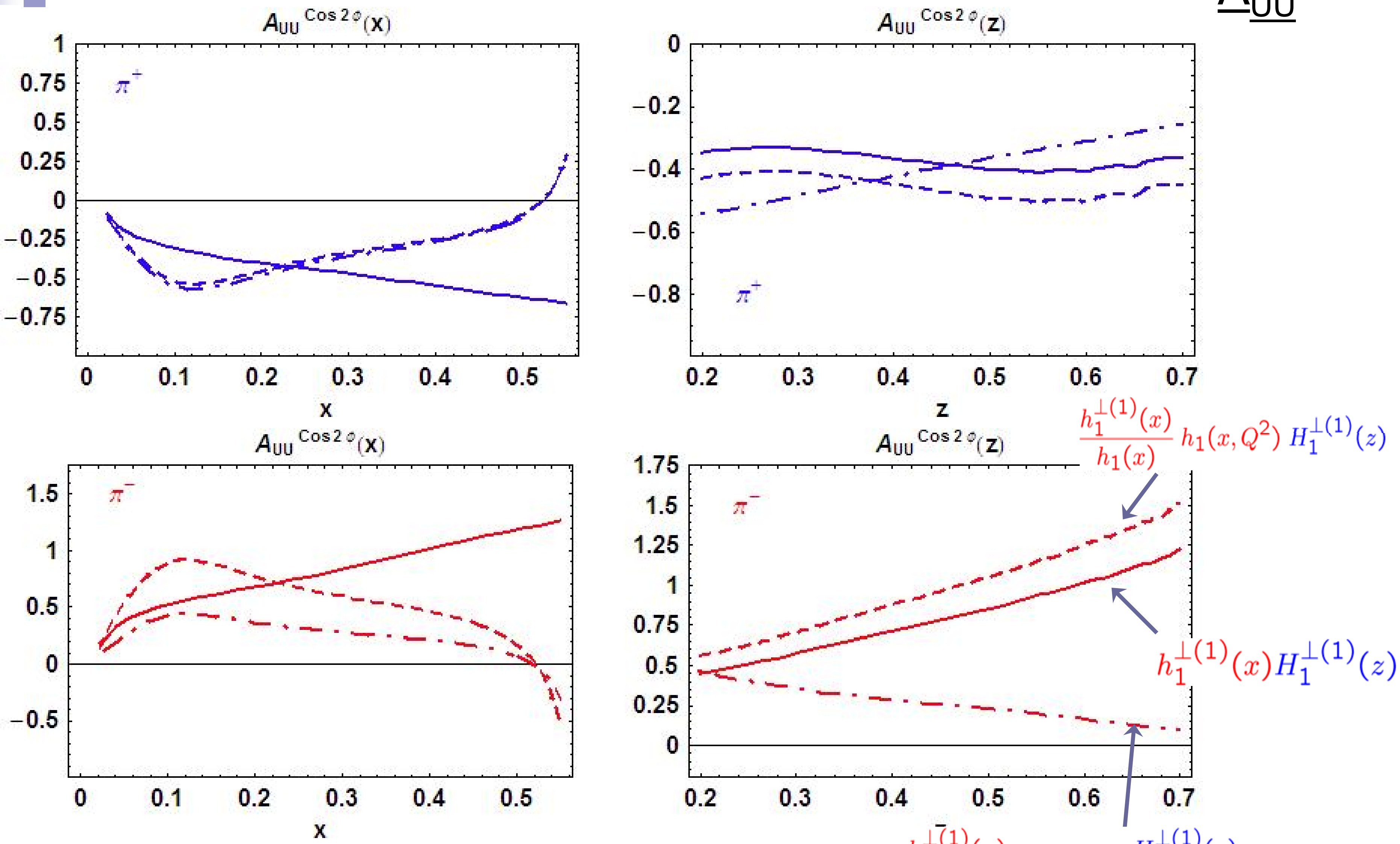
$\int d\phi_h\, d\sigma^o \propto F_{UU,T} + \epsilon\, F_{UU,L}$ + Rosenbluth separation

access to “pure Boer-Mulders” effect in cos2ϕ by two SIDIS measurements

$A_{UU}^{\cos 2\phi}$



$\frac{h_1^{\perp(1)}(x)}{h_1(x)}\, h_1(x,Q^2)\; H_1^{\perp(1)}(z)$

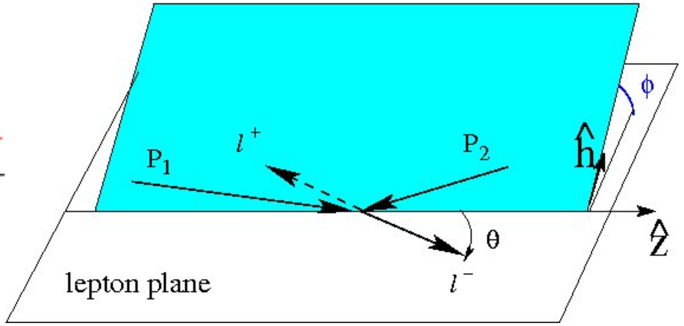
$h_1^{\perp(1)}(x) H_1^{\perp(1)}(z)$

$\frac{h_1^{\perp(1)}(x)}{h_1(x)}\, h_1(x,Q^2)\; \frac{H_1^{\perp(1)}(z)}{D_1(z)}\; D_1^{DSS}(z,Q^2)$

$$A_{UU}^{Q_T^2 \cos 2\phi_h} = 2\,\frac{B(y)}{A(y)}\,\frac{\sum_q e_q^2\, x h_1^{\perp(1)q}(x)\, H_1^{\perp(1)q}(z)}{\sum_q e_q^2\, x f_1^q(x)\, D_1^q(z)}$$

weighted SSA : Drell-Yan

$$\tilde{A}^{W}_{XY}(x_1,x_2,y) \;\propto\; \frac{\langle W\rangle_{XY}}{\langle 1\rangle_{UU}} \equiv \frac{\int d\phi_S d\phi d^2\mathbf{q}_T\, W\, d\tilde{\sigma}_{XY}}{\int d\phi_S d\phi d^2\mathbf{q}_T\, d\tilde{\sigma}_{UU}}$$



Collins-Soper frame

$$\tilde{A}_{TT} \;=\; \frac{\sin^2\theta}{1+\cos^2\theta}\,\cos(2\phi-\phi_{S_1}-\phi_{S_2})\,\frac{\sum_q e_q^2\, x_1 h_1^{\bar{q}}(x_1)\, x_2 h_1^{q}(x_2) + (\bar{q}\leftrightarrow q)}{\sum_q e_q^2\, x_1 f_1^{\bar{q}}(x_1)\, x_2 f_1^{q}(x_2) + (\bar{q}\leftrightarrow q)}$$

$$\tilde{A}_{UU}^{q_T^2\cos 2\phi} \;=\; 2\,\frac{\langle \frac{q_T^2}{4M_1M_2}\cos(2\phi)\rangle_{UU}}{\langle 1\rangle_{UU}} = 2\,\frac{\sum_q e_q^2\, x_1 h_1^{\perp(1)\bar{q}}(x_1)\, x_2 h_1^{\perp(1)q}(x_2) + (\bar{q}\leftrightarrow q)}{\sum_q e_q^2\, x_1 f_1^{\bar{q}}(x_1)\, x_2 f_1^{q}(x_2) + (\bar{q}\leftrightarrow q)}$$

$$\tilde{A}_{UT}^{q_T\sin(\phi-\phi_{S_2})} \;=\; 2\,\frac{\langle \frac{q_T}{M_2}\sin(\phi-\phi_{S_2})\rangle_{UT}}{\langle 1\rangle_{UU}} = 2\,\frac{\tilde{A}(y)}{\tilde{A}(y)}\,\frac{\sum_q e_q^2\, x_1 f_1^{\bar{q}}(x_1)\, x_2 f_{1T}^{\perp(1)q}(x_2) + (\bar{q}\leftrightarrow q)}{\sum_q e_q^2\, x_1 f_1^{\bar{q}}(x_1)\, x_2 f_1^{q}(x_2) + (\bar{q}\leftrightarrow q)}$$

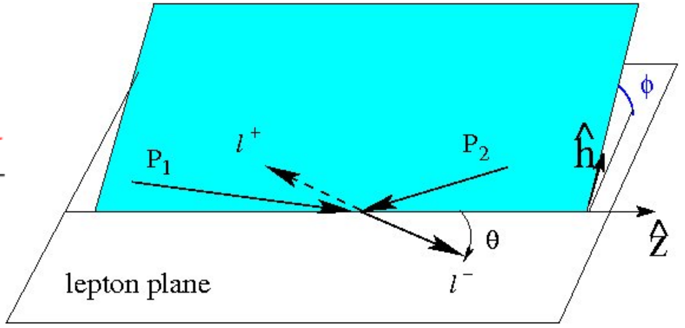
$$\tilde{A}_{UT}^{q_T\sin(\phi+\phi_{S_2})} \;=\; 2\,\frac{\langle \frac{q_T}{M_1}\sin(\phi+\phi_{S_2})\rangle_{UT}}{\langle 1\rangle_{UU}} = -2\,\frac{\tilde{B}(y)}{\tilde{A}(y)}\,\frac{\sum_q e_q^2\, x_1 h_1^{\perp(1)\bar{q}}(x_1)\, x_2 h_1^{q}(x_2) + (\bar{q}\leftrightarrow q)}{\sum_q e_q^2\, x_1 f_1^{\bar{q}}(x_1)\, x_2 f_1^{q}(x_2) + (\bar{q}\leftrightarrow q)}$$

$$\tilde{A}_{UT}^{q_T^3\sin(3\phi-\phi_{S_2})} \;=\; 2\,\frac{\langle \frac{q_T^3}{6M_1M_2^2}\sin(3\phi-\phi_{S_2})\rangle_{UT}}{\langle 1\rangle_{UU}} = -2\,\frac{\tilde{B}(y)}{\tilde{A}(y)}\,\frac{\sum_q e_q^2\, x_1 h_1^{\perp(1)\bar{q}}(x_1)\, x_2 h_{1T}^{\perp(2)q}(x_2) + (\bar{q}\leftrightarrow q)}{\sum_q e_q^2\, x_1 f_1^{\bar{q}}(x_1)\, x_2 f_1^{q}(x_2) + (\bar{q}\leftrightarrow q)}$$

$\tilde{A}(y) \;=\; (\frac{1}{2}-y+y^2) = \frac{1}{4}(1+\cos^2\theta) \quad \tilde{B}(y) = y(1-y) = \frac{1}{4}\sin^2\theta$ $\quad x_1 = \sqrt{\tau}\,e^{y}$ beam; $x_2 = \sqrt{\tau}\,e^{-y}$ target

weighted SSA : Drell-Yan

$$\tilde{A}_{XY}^{W}(x_1, x_2, y) \;\propto\; \frac{\langle W\rangle_{XY}}{\langle 1\rangle_{UU}} \equiv \frac{\int d\phi_S d\phi d^2\mathbf{q}_T\, W\, d\tilde{\sigma}_{XY}}{\int d\phi_S d\phi d^2\mathbf{q}_T\, d\tilde{\sigma}_{UU}}$$



Collins-Soper frame

$$\tilde{A}_{TT} = \frac{\sin^2\theta}{1+\cos^2\theta}\cos(2\phi-\phi_{S_1}-\phi_{S_2})\frac{\sum_q e_q^2\, x_1 h_1^{\bar{q}}(x_1)\, x_2 h_1^{q}(x_2) + (\bar{q}\leftrightarrow q)}{\sum_q e_q^2\, x_1 f_1^{\bar{q}}(x_1)\, x_2 f_1^{q}(x_2) + (\bar{q}\leftrightarrow q)}$$

$$\tilde{A}_{UU}^{q_T^2\cos 2\phi} = 2\frac{\langle \frac{\boxed{q_T^2}}{4M_1M_2}\cos(2\phi)\rangle_{UU}}{\langle 1\rangle_{UU}} = 2\frac{\sum_q e_q^2\, x_1 h_1^{\perp(1)\bar{q}}(x_1)\, x_2 h_1^{\perp(1)q}(x_2) + (\bar{q}\leftrightarrow q)}{\sum_q e_q^2\, x_1 f_1^{\bar{q}}(x_1)\, x_2 f_1^{q}(x_2) + (\bar{q}\leftrightarrow q)}$$

but later…

$$\tilde{A}_{UT}^{q_T\sin(\phi-\phi_{S_2})} = 2\frac{\langle \frac{q_T}{M_2}\sin(\phi-\phi_{S_2})\rangle_{UT}}{\langle 1\rangle_{UU}} = 2\frac{\tilde{A}(y)}{\tilde{A}(y)}\frac{\sum_q e_q^2\, x_1 f_1^{\bar{q}}(x_1)\, x_2 f_{1T}^{\perp(1)q}(x_2) + (\bar{q}\leftrightarrow q)}{\sum_q e_q^2\, x_1 f_1^{\bar{q}}(x_1)\, x_2 f_1^{q}(x_2) + (\bar{q}\leftrightarrow q)}$$

$$\tilde{A}_{UT}^{q_T\sin(\phi+\phi_{S_2})} = 2\frac{\langle \frac{q_T}{M_1}\sin(\phi+\phi_{S_2})\rangle_{UT}}{\langle 1\rangle_{UU}} = -2\frac{\tilde{B}(y)}{\tilde{A}(y)}\frac{\sum_q e_q^2\, x_1 h_1^{\perp(1)\bar{q}}(x_1)\, x_2 h_1^{q}(x_2) + (\bar{q}\leftrightarrow q)}{\sum_q e_q^2\, x_1 f_1^{\bar{q}}(x_1)\, x_2 f_1^{q}(x_2) + (\bar{q}\leftrightarrow q)}$$

$$\tilde{A}_{UT}^{q_T^3\sin(3\phi-\phi_{S_2})} = 2\frac{\langle \frac{\boxed{q_T^3}}{6M_1M_2^2}\sin(3\phi-\phi_{S_2})\rangle_{UT}}{\langle 1\rangle_{UU}} = -2\frac{\tilde{B}(y)}{\tilde{A}(y)}\frac{\sum_q e_q^2\, x_1 h_1^{\perp(1)\bar{q}}(x_1)\, x_2 h_{1T}^{\perp(2)q}(x_2) + (\bar{q}\leftrightarrow q)}{\sum_q e_q^2\, x_1 f_1^{\bar{q}}(x_1)\, x_2 f_1^{q}(x_2) + (\bar{q}\leftrightarrow q)}$$

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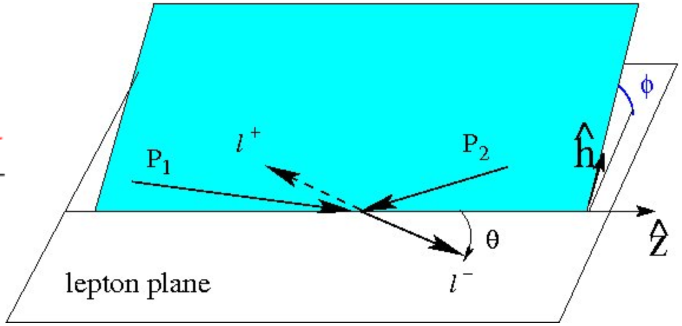
$x_1 = \sqrt{\tau}\,e^{y}$ beam; $x_2 = \sqrt{\tau}\,e^{-y}$ target

weighted SSA : Drell-Yan

$$\tilde{A}^{W}_{XY}(x_1,x_2,y) \;\propto\; \frac{\langle W\rangle_{XY}}{\langle 1\rangle_{UU}} \equiv \frac{\int d\phi_S d\phi d^2\mathbf{q}_T \, W \, d\tilde{\sigma}_{XY}}{\int d\phi_S d\phi d^2\mathbf{q}_T \, d\tilde{\sigma}_{UU}}$$



Collins-Soper frame

a$_{TT}$

$$\tilde{A}_{TT} = \frac{\sin^2\theta}{1+\cos^2\theta}\cos(2\phi-\phi_{S_1}-\phi_{S_2}) \frac{\sum_q e_q^2\, x_1 h_1^{\bar{q}}(x_1)\, x_2 h_1^q(x_2) + (\bar{q}\leftrightarrow q)}{\sum_q e_q^2\, x_1 f_1^{\bar{q}}(x_1)\, x_2 f_1^q(x_2) + (\bar{q}\leftrightarrow q)}$$

$$\tilde{A}_{UU}^{q_T^2 \cos 2\phi} = 2\frac{\langle \frac{q_T^2}{4M_1M_2}\cos(2\phi)\rangle_{UU}}{\langle 1\rangle_{UU}} = 2\frac{\sum_q e_q^2\, x_1 h_1^{\perp(1)\bar{q}}(x_1)\, x_2 h_1^{\perp(1)q}(x_2) + (\bar{q}\leftrightarrow q)}{\sum_q e_q^2\, x_1 f_1^{\bar{q}}(x_1)\, x_2 f_1^q(x_2) + (\bar{q}\leftrightarrow q)}$$

but later…

$$\tilde{A}_{UT}^{q_T \sin(\phi-\phi_{S_2})} = 2\frac{\langle \frac{q_T}{M_2}\sin(\phi-\phi_{S_2})\rangle_{UT}}{\langle 1\rangle_{UU}} = 2\frac{\tilde{A}(y)}{\tilde{A}(y)}\frac{\sum_q e_q^2\, x_1 f_1^{\bar{q}}(x_1)\, x_2 f_{1T}^{\perp(1)q}(x_2) + (\bar{q}\leftrightarrow q)}{\sum_q e_q^2\, x_1 f_1^{\bar{q}}(x_1)\, x_2 f_1^q(x_2) + (\bar{q}\leftrightarrow q)}$$

all prediction

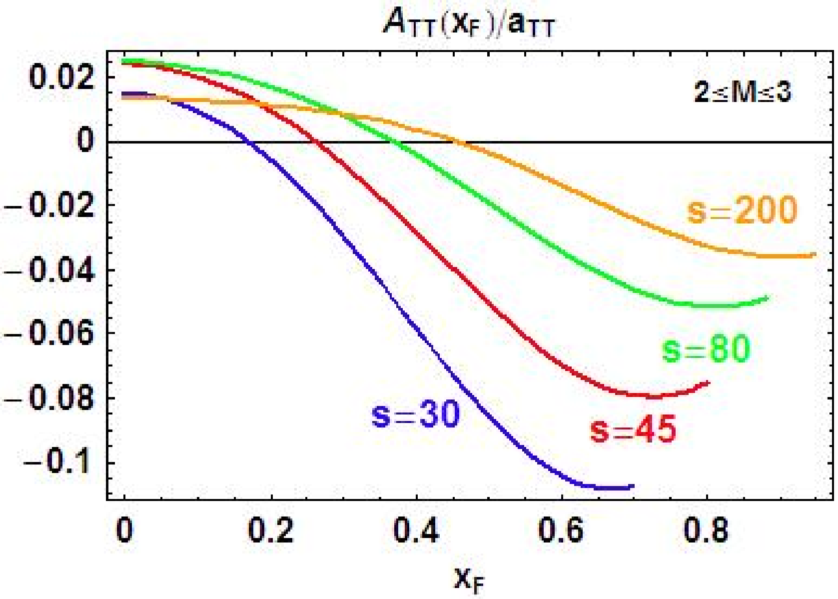
$$\tilde{A}_{UT}^{q_T \sin(\phi+\phi_{S_2})} = 2\frac{\langle \frac{q_T}{M_1}\sin(\phi+\phi_{S_2})\rangle_{UT}}{\langle 1\rangle_{UU}} = -2\frac{\tilde{B}(y)}{\tilde{A}(y)}\frac{\sum_q e_q^2\, x_1 h_1^{\perp(1)\bar{q}}(x_1)\, x_2 h_1^q(x_2) + (\bar{q}\leftrightarrow q)}{\sum_q e_q^2\, x_1 f_1^{\bar{q}}(x_1)\, x_2 f_1^q(x_2) + (\bar{q}\leftrightarrow q)}$$

$$\tilde{A}_{UT}^{q_T^3 \sin(3\phi-\phi_{S_2})} = 2\frac{\langle \frac{q_T^3}{6M_1M_2^2}\sin(3\phi-\phi_{S_2})\rangle_{UT}}{\langle 1\rangle_{UU}} = -2\frac{\tilde{B}(y)}{\tilde{A}(y)}\frac{\sum_q e_q^2\, x_1 h_1^{\perp(1)\bar{q}}(x_1)\, x_2 h_{1T}^{\perp(2)q}(x_2) + (\bar{q}\leftrightarrow q)}{\sum_q e_q^2\, x_1 f_1^{\bar{q}}(x_1)\, x_2 f_1^q(x_2) + (\bar{q}\leftrightarrow q)}$$

$$\tilde{A}(y) = (\tfrac{1}{2}-y+y^2) = \tfrac{1}{4}(1+\cos^2\theta) \qquad \tilde{B}(y) = y(1-y) = \tfrac{1}{4}\sin^2\theta$$

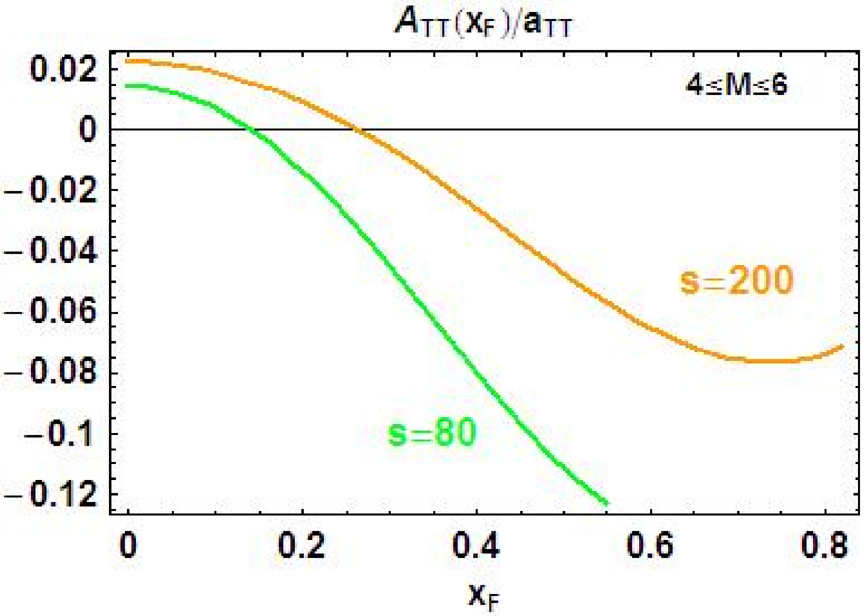
$$x_1 = \sqrt{\tau}\, e^{y} \text{ beam; } x_2 = \sqrt{\tau}\, e^{-y} \text{ target}$$

$\tilde{A}_{TT}$



$$\bar{p}^{\uparrow} p^{\uparrow} \to \mu^+ \mu^- X$$

fixed target (PANDA)		collider (PAX?)
s=30	→	200 GeV^2
0.1< τ < 0.3	→	0.02< τ < 0.045



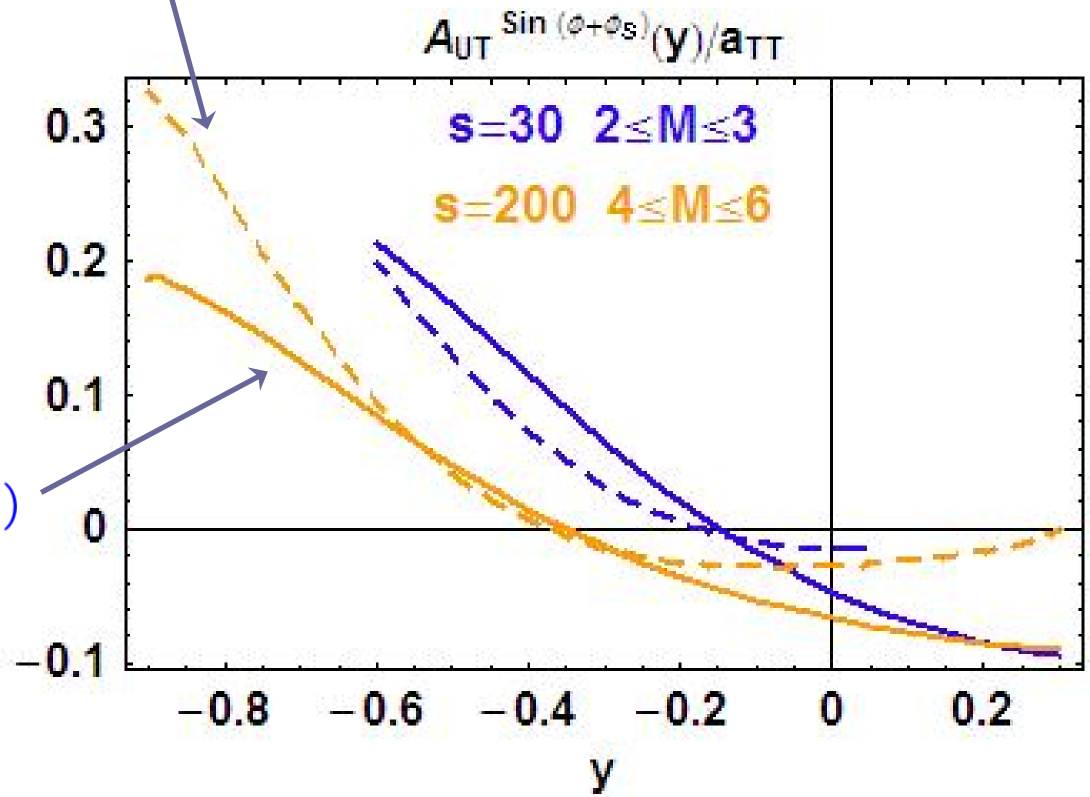
$$\tau = \frac{M^2}{s} \; ; \; x_F = x_1 - x_2$$

$\underline{\tilde{A}_{UT}^{\sin(\phi + \phi s)}}$

$\bar{p}p^{\uparrow} \longrightarrow \mu^{+}\mu^{-}X$

fixed target (PANDA)	collider (PAX?)	
s=30	200	GeV²
0.1< τ < 0.3	0.02< τ < 0.045	

$\frac{h_1^{\perp(1)}(x_1)}{h_1(x_1)}\, h_1(x_1, Q^2)\, h_1(x_2)$



$h_1^{\perp(1)}(x_1) h_1(x_2)$

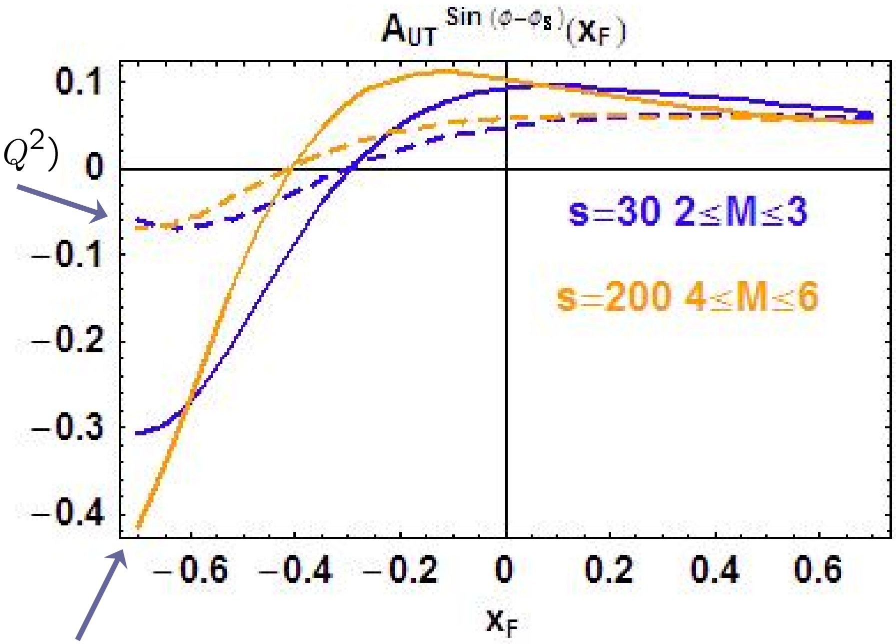
$$\begin{aligned} \tau &= \frac{M^2}{s} \\ x_F &= x_1 - x_2 \\ y &= \tfrac{1}{2}\log\frac{x_1}{x_2} \end{aligned}$$

$$\tilde{A}_{UT}^{q_T\,\sin(\phi+\phi_{S_2})} = -2\,\frac{\tilde{B}(y)}{\tilde{A}(y)}\,\frac{\sum_q e_q^2\, x_1 h_1^{\perp(1)\bar{q}}(x_1)\, x_2 h_1^q(x_2) + (\bar{q} \leftrightarrow q)}{\sum_q e_q^2\, x_1 f_1^{\bar{q}}(x_1)\, x_2 f_1^q(x_2) + (\bar{q} \leftrightarrow q)}$$

$$\underline{\tilde{A}_{UT}^{\sin(\phi - \phi s)}}$$

$$\bar{p}p^{\uparrow} \to \mu^{+}\mu^{-}X$$

fixed target (PANDA)	collider (PAX?)	
s=30	200	GeV^2
0.1< τ < 0.3	0.02< τ < 0.045	



$$\frac{f_{1T}^{\perp(1)}(x)}{f_1(x)}\, f_1(x,Q^2)$$

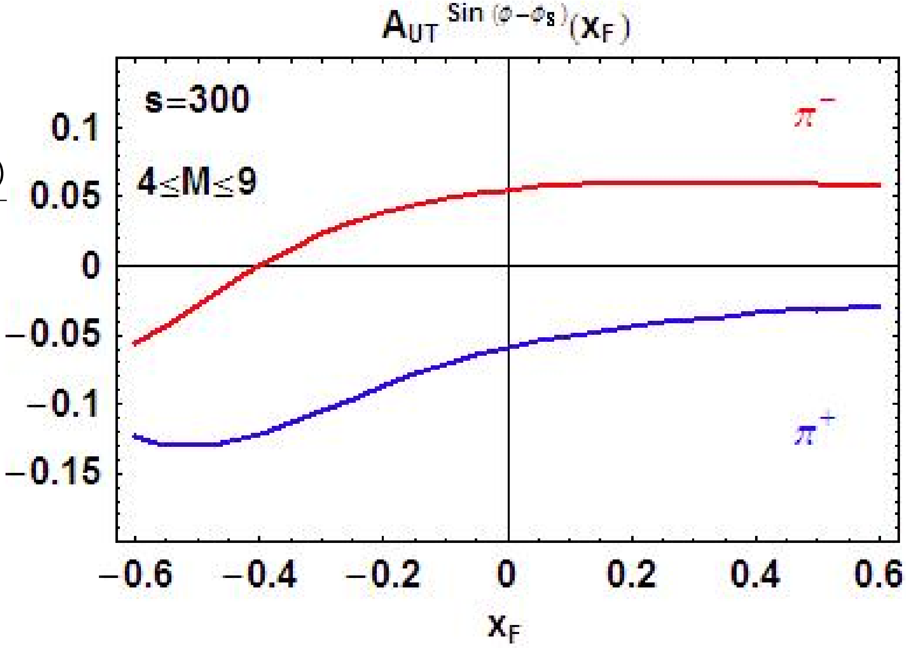
$$f_{1T}^{\perp(1)}(x)$$

$$\tilde{A}_{UT}^{q_T \sin(\phi-\phi_{S_2})} = 2\,\frac{\tilde{A}(y)}{\tilde{A}(y)}\, \frac{\sum_q e_q^2\, x_1 f_1^{\bar{q}}(x_1)\, x_2 f_{1T}^{\perp(1)q}(x_2) + (\bar{q} \leftrightarrow q)}{\sum_q e_q^2\, x_1 f_1^{\bar{q}}(x_1)\, x_2 f_1^{q}(x_2) + (\bar{q} \leftrightarrow q)}$$

$$\tau = \frac{M^2}{s}\ ;\ x_F = x_1 - x_2$$

$$\pi^{-}p^{\uparrow} \to \mu^{+}\mu^{-}X$$

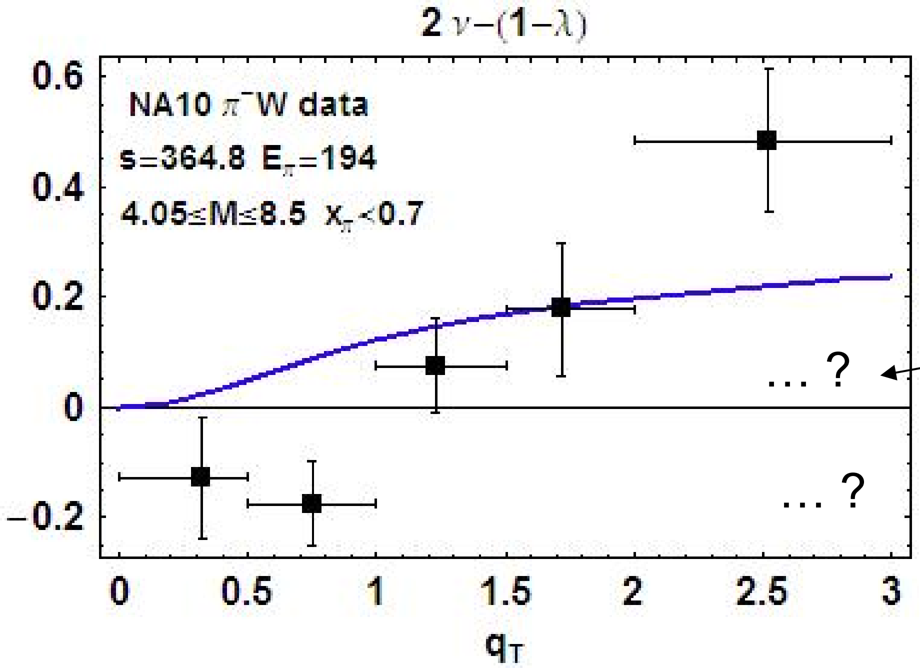
fixed target
COMPASS
s=300 GeV^2
0.05< τ < 0.27



unweighted $\tilde{A}_{UU}^{\cos 2\phi}$

$$\pi^- p \to \mu^+ \mu^- X$$

$$\frac{1}{\sigma}\frac{d\sigma}{d\Omega} = \frac{3}{4\pi}\frac{1}{\lambda+3}\left[1+\lambda\cos^2\theta+\mu\sin^2\theta\cos\phi+\frac{\nu}{2}\sin^2\theta\cos 2\phi\right]+o(\alpha_s)$$



NLO pQCD

parton model

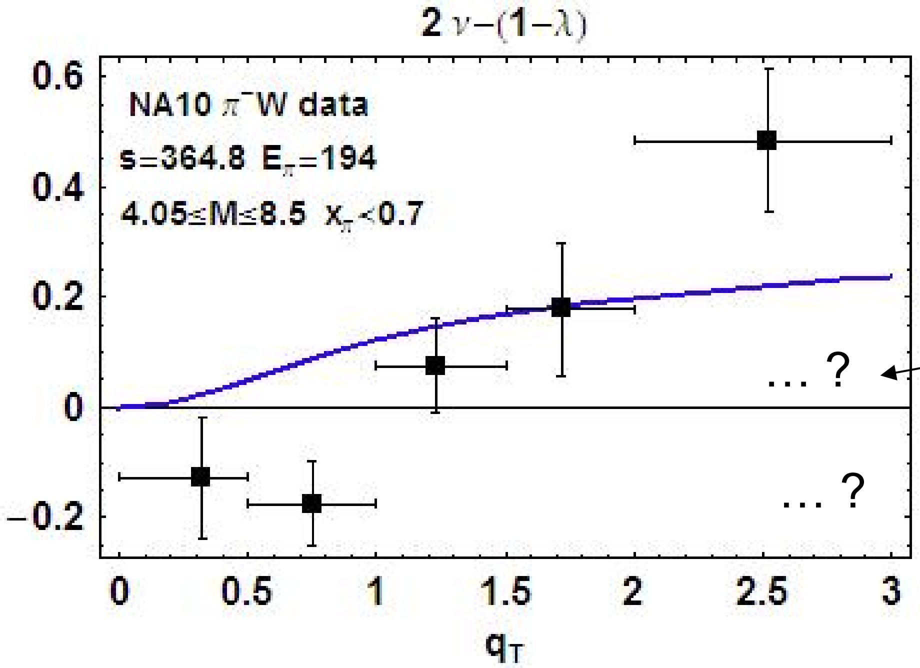
LO pQCD

unweighted $\tilde{A}_{UU}^{\cos 2\phi}$

$$\frac{1}{\sigma}\frac{d\sigma}{d\Omega} = \frac{3}{4\pi}\frac{1}{\lambda+3}\left[1+\lambda\cos^2\theta+\mu\sin^2\theta\cos\phi+\frac{\nu}{2}\sin^2\theta\cos 2\phi\right]+o(\alpha_s)$$

$$\pi^- p \rightarrow \mu^+\mu^- X$$

$$\underset{\displaystyle 2\nu}{\underset{\downarrow}{\tilde{A}_{UU}^{\cos 2\phi}}} = \underset{\displaystyle (1-\lambda)}{\underset{\downarrow}{\tilde{A}_{UU}^{\cos 2\phi}\big|_{pQCD}}} + \tilde{A}_{UU}^{\cos 2\phi}\big|_{BM}$$



NLO pQCD

parton model

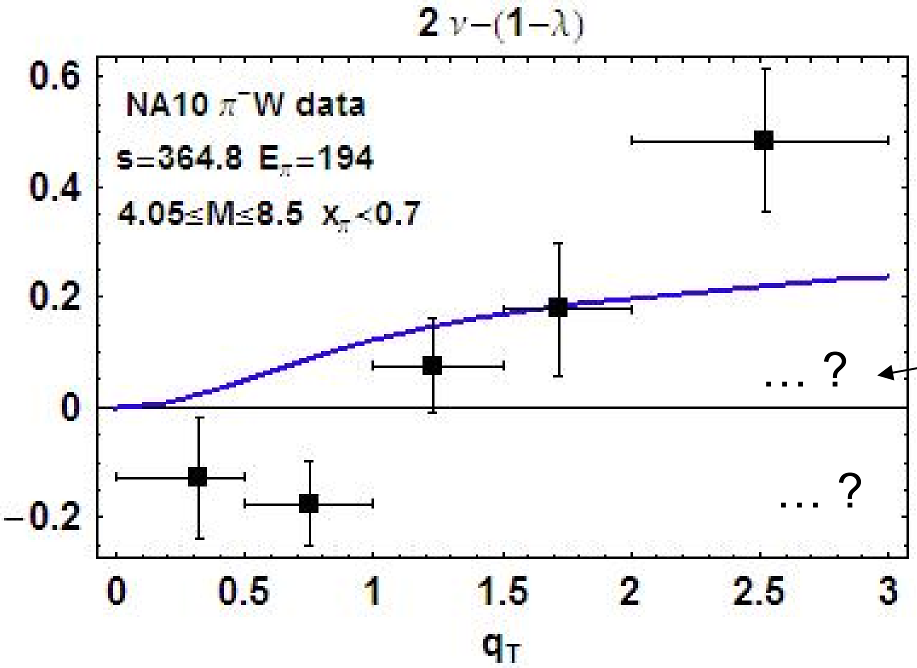
LO pQCD

unweighted $\tilde{A}_{UU}^{\cos2\phi}$

$$\frac{1}{\sigma}\frac{d\sigma}{d\Omega}=\frac{3}{4\pi}\frac{1}{\lambda+3}\left[1+\lambda\cos^2\theta+\mu\sin^2\theta\cos\phi+\frac{\nu}{2}\sin^2\theta\cos 2\phi\right]+o(\alpha_s)$$

$$\pi^- p \to \mu^+\mu^- X$$

$$\underset{\substack{\downarrow\\ 2\nu}}{\tilde{A}_{UU}^{\cos 2\phi}} = \underset{\substack{\downarrow\\ (1-\lambda)}}{\tilde{A}_{UU}^{\cos 2\phi}\big|_{pQCD}} + \tilde{A}_{UU}^{\cos 2\phi}\big|_{BM}$$



NLO pQCD

parton model

LO pQCD

- valence approximation in annihilations: $\pi^- p \approx \bar{u}u$
- "universal" ratio (see Boer, P.R.D**60**,014012 (99)): $\frac{h_1^{\perp\,\bar{u}/\pi^-}}{f_1^{\bar{u}/\pi^-}} \approx \frac{h_1^{\perp\,\bar{u}/\bar{p}}}{f_1^{\bar{u}/\bar{p}}}$
- solve analytically the convolution

$$\frac{\mathcal{C}\left[w\,\frac{h_1^{\perp\bar{u}/\bar{p}}(x_1,p_{T1})}{M_1}\,\frac{h_1^{\perp u/p}(x_2,p_{T2})}{M_2}\right]}{\mathcal{C}\left[f_1^{\bar{u}/\bar{p}}(x_1,p_{T1})\,f_1^{u/p}(x_2,p_{T2})\right]}$$

$$\begin{aligned} w &= 2\hat{\mathrm{h}}\cdot\mathrm{p}_{T1}\hat{\mathrm{h}}\cdot\mathrm{p}_{T2}-\mathrm{p}_{T1}\cdot\mathrm{p}_{T2} \\ \hat{\mathrm{h}} &= \mathrm{q}_T/|\mathrm{q}_T| \end{aligned}$$

Conclusions

- why another model for **TMD** ? We actually don't know much about them…

- why a spectator diquark model ? It's simple: always analytic results

  why including axial-vector diquarks? Necessary for down quarks, but need to improve

- what's new in our work ?
  - systematic calculation of all T-even and T-odd **TMD**
  - several forms for N-q-Dq vertex and spin=1 Dq propagator $d^{\mu\nu}$ explored
  - fix 9 parameters by fitting $f_1^{u,d}(x)$, $g_1^{u,d}(x)$ at low scale $\Rightarrow$ model scale Q_0^2=0.3 GeV^2
  - **TMD** as overlaps of lcwf ; orbital $L_{q\text{-}Dq} \neq 0$ in g.s. of N $\Rightarrow$ ~~SU(4)~~ N w.f.

results : non-gaussian, unfactorized, flavor-dependent $\mathbf{p}_T$ dependence
g.s. of N with $L_{q\text{-}Dq} \neq 0$
encouraging comparison with available parametrizations and weighted SSA
(model-independent analysis)

- future : calculate (numerically and, when possible, analytically) unweighted SSA
  add sea quarks → improve on down quark
  ….